THOMAS MASTERSON HIS THIRD BOOKE OF ARITHMETICKE.

Shewing the more ingenious inuentions, and the figuratiue and caracterical operations, by which to calculate the true ſolutions or anſwers of Arithmeticall queſtions: after a more perfect, plaine, briefe, and well ordered arithmetical way then any other heretofore publiſhed. No leſſe neceſſary and pleaſant to all men employed in great and worthie actions, then to others.

Nothing without labour,
All things with reaſon.

Imprinted at London by Richard Field.
1595.

TO THE RIGHT HONORABLE SIR IOHN PVCKERING KNIGHT, LORD KEEPER OF THE GREAT SEALE OF ENGLAND, AND ONE OF THE Lords of her Maiesties moſt honorable Priuie Counſell, THOMAS MASTERSON wiſheth increaſe of honor, and after this life endleſſe felicitie.

RIght Honourable, in the beginning the Omnipotent and euer liuing God (great architector of all things) out of a heauy lumpe or diſordred chaos, *framed the heauens, the earth, the ſea, and all things in them contayned, in number, weight and meaſure: obſeruing ſuch an harmonie therein, that the Sunne, Moone, and ſtarres, which he placed aboue (as it were the vault or couerture of his worke) ſerue for lightes, ſignes, ſeaſons, yeares, and dayes, to grace, gouerne, and direct beneath. All which by him ſo wrought, he commaunded to keepe their appointed place, order, and courſe, which they accordingly obey, and accompliſh, in ſuch wonderfull manner, that by the ſight & conſideration of them, all nations are enforced to confeſſe, that the heauens declare the inexpiicable great power and glorie of God. So that this incomprehenſible worke, cauſeth the wiſer ſort of men (Philoſophers labouring no doubt by the inſpiration of God, to find out the nature and effect of things) to conclude ſuch a* ſympathy *and agreement between man (whom they call the leſſer world) and the great frame of the world in the which we are wrapt, that there is not any motion in the heauenly bodies, whoſe operation man doth not feele in him ſelfe at the ſame inſtant. In like*

manner, it causeth Phisitions and chirurgians (seeming to maintaine the Philosophers opinion) to affirme that the places, aspectes and influences of planets, and other starres, are greatly to be obserued in letting of bloud, making incisions, working of cures, and ministring of medicines. Sea-men also and nauigators (which see the wonders of God) acknowledge by the Almighties fauour and permission, voyages to be accomplished, through the obseruation of the heauens orders, and the things in them contained. To be briefe, it causeth the husbandmen (which labour to haue increase of things conuenient for mans vse) to say, that the reuolutions of the heauens, do as well tell them their due seasons to sowe, plant, graffe, geld, &c. as cause the earth to produce her fruites to the desired end or harueſt. And generally causeth all men (as they differ more or lesse from brute beastes) to speake continually of the great benefites they receiue of almightie God. But (right Honourable) seeing God made, gouerneth, and maintaineth all things in number, weight and measure, it is verie difficult for man to know any thing certainly concerning the celestiall spheres, or assuredly to speake and determine, of terrestriall and humane affaires, without that excellent gift of God the science of numbers. Therefore (according to the small measure of knowledge which God hath giuen me therein) I haue proceeded to publish this my third booke of Arithmeticke, treating of the harder arithmeticall operations (omitted in my first and second bookes) thereby to calculate the answers of arithmeticall questions, of celestiall, terrestriall and marine things, past, present, and to come, as well in times of warre as peace. The which as a token of my good will and dutifull seruice, in all humble manner I deducate vnto your Lordships protection and patronage.

Your Lordships most humble
and dutifull seruant:

Thomas Masterson.

To

To the Reader.

HAVING (gentle Reader) by my two former bookes in ſome ſort prepared the minds of thoſe which vnderſtand them aright, and in a manner enabled them to conceiue greater things: I do now next in order produce this my third booke; wherein thou haſt the definitions with their declarations, and the inſtructions with their examples, that are expedient and needfull to be had and perfectly knowen, for the further attaining to the true vnderſtanding of this Art. And albeit that they are briefe and few in number, yet they containe much, and giue light to thoſe hardeſt places of *Euclids Elements* which haue alwayes bene taken to be very difficult, interchangeably helping to vnderſtand each other. So that by the true and artificiall vſe of them, many ſtrange, profitable, and worthie actions are accompliſhed: as by thy working thou mayeſt know, if thou doeſt attaine to the right vnderſtanding of them and their applications. And for thy order of proceeding herein, my counſell is (as it hath bene in my other bookes) that thou forſake the bare imitating of rules: for manie times when a queſtion is propounded to be anſwered, thou art brought by them, as it were in a maze, not knowing which way to take, nor what rule to follow, remaining vncertaine with truth how to worke. Therefore labour diligently to know the liuely reaſon and naturall effect of things: for thereby thou ſhalt profit in the ſtudie of this Art, increaſing thy skill according to thine owne deſire. Thus hoping that my labour and counſell will be acceptable to thee, and to haue the faultes eſcaped friendly amended, I bidde thee moſt heartely farewell.

From London, anno 1595.

The firſt definition.
What an euen number is. Fol.
The ſecond definition.
What an odde number is. 1
The third definition.
What a compound number is. 1
The fourth definition.
What an vncompound number is. 2
The fift definition.
What a perfect number is. 2
The ſixt definition.
What a Surd number is. 2
The ſeuenth definition.
Which numbers are communicants. 3
The eight definition.
Which numbers are incommunicants. 3
The ninth definition.
Which numbers are in proportion continuall. 3
The tenth definition.
What number is the middle proportionall betweene two numbers. 3
The eleuenth definition.
What a ſquare number is. 3
The twelfth definition.
What a cube number is. 4
The thirteenth definition.
Which numbers are in progreſſion arithmeticall. 4
The fourteenth definition.
Which numbers are in progreſſion Geometricall. 4
The fifteenth definition.
Which numbers are in progreſſion harmonicall. 4
The firſt inſtruction.
Of ſignes and caracters, and their ſignifications or numeration. 5
The ſecond inſtruction.
Of the extraction of the ſquare roote, cubicke roote, zenſezenſicke roote, ſurſolide roote, or any other roote, many different wayes. 7

The third inſtruction.

Of the extraction of any roote in fractions. Fol. 30

The fourth inſtruction.

Of Surd numbers, and how they are expreſſed and pronounced: alſo how to finde the value of them, ſo nigh as ſhall be thought neceſſarie. 31

The fift inſtruction.

Of abbreuiation in Surd mediall numbers. 34

The ſixt inſtruction.

Of bringing or reducing two Surd mediall numbers giuen, vnder one like name of caracter. 34

The ſeuenth inſtruction.

To finde the reaſon betweene two Surd mediall numbers giuen of one kinde, compared together. 36

The eight inſtruction.

Of Addition in Surd mediall numbers. 36

The ninth inſtruction.

Of Subſtraction in Surd mediall numbers. 38

The tenth inſtruction.

Of Multiplication in Surd mediall numbers. 38

The eleuenth inſtruction.

Of Deuiſion in Surd mediall numbers. 39

The twelfth inſtruction.

Of Addition of the ſignes + and —. 40

The thirteenth inſtruction.

Of Subſtraction of the ſignes + and —. 40

The fourteenth inſtruction.

Of Multiplication of the ſignes + and —. 41

The fifteenth inſtruction.

Of Deuiſion of the ſignes + and —. 43

The ſixteenth inſtruction.

Of Addition in ioyned Surd numbers. 43

The ſeuenteenth inſtruction.

Of Subſtraction in ioyned Surd numbers. 44

The eighteenth inſtruction.

Of Multiplication in ioyned Surd numbers. 45

The nineteenth inſtruction.

Of Deuiſion in ioyned Surd numbers. Fol.48

The twentieth inſtruction.

Of Addition in diſtinct Surd numbers. 51

The one and twentieth inſtruction.

Of Subſtraction in diſtinct Surd numbers. 52

The two and twentieth inſtruction.

Of Multiplication in diſtinct Surd numbers. 53

The three and twentieth inſtruction.

Of Deuiſion in diſtinct Surd numbers. 55

The foure and twentieth inſtruction.

Of Addition of Reaſons. 56

The fiue and twentieth inſtruction.

Of Subſtraction of Reaſons. 57

The ſixe and twentieth inſtruction.

Of Multiplication of Reaſons. 58

The ſeuen and twentieth inſtruction.

Of Deuiſion of Reaſons. 59

The eight and twentieth inſtruction.

To finde betweene two numbers giuen of one kind, ſo many numbers middle proportionals as is required. 61

The nine and twentieth inſtruction.

Of extractions of rootes in Surd numbers, by diuerſe ſeuerall wayes. 62.

THO-

THOMAS MASTERSON HIS THIRD BOOKE OF ARITHMETICKE.

Definitions.

The first Definition.

EVen nūbers are, thoſe which may be parted into two equall parts.

Declaration.

That multitude of things of one kind, which may be parted into two ſuch parts, that the one may iuſt containe ſo many of thoſe things, as the other part doth, and neither more nor leſſe, without deuiding the thing to the vnit limited; is called an euen number, as 2. 4. 6. 8. 10. 12. &c. infinitely.

The 2 Definition.

Odde numbers are, thoſe which cannot be parted into two equall parts.

Declaration.

This is the conuerſe of the aforegoing, and by the declaratiō of the ſame it is apparant, that an odde number can not be parted into two equal parts, without deuiding the thing to the vnit limited. Alſo that euery euen number, hauing one vnit more added vnto it, is an odde number, as 3. 5. 7. 9. 11. 13. &c. infinitely.

The 3 Definition.

Compound numbers are, thoſe which may be meaſured by another number.

Declaration.

That number which doth containe another number, ſo many

times as a third number doth containe vnits, is called a cõpound number, and is compounded of that ſayd other number which it doth containe, and of that third number which doth containe ſo many aforeſaid vnits. Alſo by the third definition of my firſt book, the number meaſuring another number, is a part therof: & by the 5 definition of my firſt booke, the number meaſured is, the many times of the number meaſuring. Then the number meaſured is compounded of the number meaſuring, and the number of the ſo many times, as the number meaſured doth cõtaine the number meaſuring: for each of them taken ſo manie times as the other doth containe vnits, doth produce the ſame. As 12 is compounded of 3 and 4: or of 2 and 6 &c.

The 4 Definition.

Vncompound numbers are thoſe, which cannot be meaſured by another number.

Declaration.

Then thoſe numbers, which are not the many times of another number, without deuiding the thing to the vnit limited, are called vncompound numbers, as, 2. 3. 5. 7. 11. 13. 17. &c.

The 5 Definition.

Perfect numbers are thoſe, which are equall to the aggregat of all their parts.

Declaration.

By the firſt definition of my firſt booke, vnit is that by which euery thing that is, is ſayd one: ſo number in theſe fiue definitions muſt be taken for a multitude of thoſe things to the vnit firſt limited; alſo the parts of number, for a multitude of vnits, and the leaſt part of number for one vnit. As the parts of 6 are 3 2 and 1: which added together, their aggregat is 6: therefore 6. 28. 496. 8128. &c. are called perfect numbers.

The 6 Definition.

Surd numbers are, the ſignifications or repreſentations of irrationall, vnpronounceable or ſurd quantities.

Declaration.

In the 6 inſtruction of my firſt booke we ſhewed, the cauſe why that a ſecond diſtinct poſition and conſideration of num-

bers,

bers, is called a fraction, or broken number: and nowe for the vnderſtanding of the reaſon why this third diſtinct poſition and conſideration of number, is called a ſurd number, we haue written the fourth inſtruction of this booke, which doth declare the ſame at large, therefore to auoyd vaine repetitions, we referre thee to that place.

The 7 Definition.

Thoſe numbers are communicants whoſe reaſon is rationall.

Declaration.

Thoſe numbers whoſe reaſon may be expreſſed and named by number, either whole or broken, are here called communicants.

The 8 Definition.

Thoſe numbers are incommunicants whoſe reaſon is ſurd.

Declaration.

This is the conuerſe of the afore going, and by the 4 and 7 inſtructions of this booke, theſe two definitions are made apparant, therefore we referre thee to thoſe places for the better vnderſtanding of them.

The 9 Definition.

Thoſe numbers are in proportion continuall, where the firſt hath ſuch reaſon to the ſecond, as the ſecond hath to the third, and the third to the fourth, &c.

Declaration.

As 1. 2. 4. 8. 16. &c. or 81. 27. 9. 3. 1: or 16. 4. 1. $\frac{1}{4}$. $\frac{1}{16}$. $\frac{1}{64}$. &c. infinitely: and the like in ſurd numbers, &c.

The 10 Definition.

A number is the middle proportionall betweene two numbers, when the one of them hath the ſame reaſon to it, that it hath to the other.

Declaration.

As 6 is the middle proportional betweene 4 and 9: or $1\frac{1}{2}$. betweene $\frac{1}{4}$ and 3 &c. infinitely: and the like in ſurd numbers. &c.

The 11 Definition.

A ſquare number, is the product of a number two times taken and multiplied: that is, the product of a number multi-

plied by the ſame number.

Declaration.

As 4, produced of 2 multiplied by 2. or 169, produced of 13 multiplied by 13. or $2\frac{1}{4}$, produced of $1\frac{1}{2}$ multiplied by $1\frac{1}{2}$ &c. infinitely: and the like in ſurd numbers, &c.

The 12 Definition.

A cube number is, the product of a number three times taken and multiplied: that is the product of a number multiplied by the ſquare number of the ſame number.

Declaration.

As 8, produced of the product of 2 multiplied by 2 multiplied by 2: that is of 4 multiplied by 2. &c. infinitely: and the like in fractions and ſurd numbers, &c.

The 13 Definition.

Thoſe numbers are in Progreſſion arithmeticall, which are ſo rehearſed or placed one after other, that the firſt is ſo much greater or leſſer then the ſecond, as the ſecond is then the third, and the third then the fourth, and ſo following to the ende in one exceſſe continuall.

Declaration.

As 1. 2. 3. 4. &c. or 8. 6. 4. 2. and the like, in fractions and ſurd numbers &c.

The 14 Definition.

Thoſe numbers are in Progreſſion Geometricall, which are ſo rehearſed or placed one after other, that the firſt hath ſuch reaſon to the ſecond, as the ſecond hath to the third, and the third to the fourth, and ſo continuing to the end in proportion continuall.

Declaration.

As 1. 2. 4. 8. &c. or 27. 9. 3. 1. and the like, in fractions and ſurd numbers, &c.

The 15 Definition.

Thoſe numbers are in Progreſſion harmonicall, which are ſo rehearſed or placed one after other, that the exceſſe betweene the firſt and the ſecond, to the exceſſe betweene the ſecond and the third, hath one and the ſame reaſon, that the firſt hath to the

the third. Againe, the exceſſe betweene the ſecond & the third, to the exceſſe betweene the third and the fourth, hath one and the ſame reaſon that the ſecond hath to the fourth: and ſo following to the end.

Declaration.

As 3. 4. 6. or 6. 8. 12. 24. or 60. 70. 84. 105. 140. 210. 420. &c. and the like, in fractions and ſurd numbers, &c.

There are many other variations in Muſicke, which at this time we omit. Thus in this maner, for the better attaining to the purpoſe pretended, it is conuenient to vnderſtand number in the 1. 2. 3. 4. and 5. definitions in one nature, and in the 6. 7. 8. 9. 10. 11. 12. 13. 14. and 15. definitions (abuſiuely) in another nature, &c.

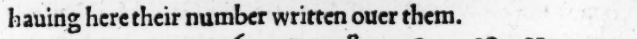
INSTRVCTIONS.

The firſt Inſtruction is of ſignes and Caracters, and their ſignifications.

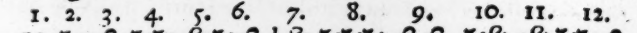
THe chiefe ende looked for in this art, is to find, manifeſt, expreſſe, & name, the deſired vnknowne quantitie. And for the attaining thereunto, beſide all the figures, inuentions and operations, ſhewed and declared in my firſt and ſecond bookes: we vſe this ſigne + for this word *more*, & this ſigne — for this word *leſſe*. And againe for breuitie ſake, we call and name abuſiuely the vnit, number, fraction, and ſurd quantitie, generally number; but with this conſideration, that we haue a ſpeciall regard, what is meant and ſignified by them. And moreouer, whereas any number firſt giuen, taken, or imagined, may be multiplied, or imagined to be multiplied by it ſelf, and that product againe by the ſame firſt number, and then that product againe by the ſame firſt number, and ſo euer the laſt product by the ſame firſt number infinitely: thoſe ſayd products, and the number firſt giuen taken or imagined, we ſignifie and diſcerne one from other by theſe caracters following ſimply placed before them, yet

hauing here their number written ouer them.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12.



13. 14. 15.



The firſt caracter written thus ⅋, doeth ſignifie any number before which it is written, to be the firſt number giuen, taken, or imagined; and is called *radix*, or roote, for that all the other caracters haue their originall or of-ſpring of it. The ſecond written thus ʒ, is called *zenſe* or ſquare, and doth ſignifie any number before which it is written, to be the product of the firſt multiplication of the roote by it ſelfe: that is of the roote two times taken and multiplied. The third written thus ↄ, is called *cube*, and doth ſignifie the number following the ſame to be the product of the ſecond multiplication of the roote, three times taked and multiplied, that is of the ʒ multiplied by the ⅋. The fourth is called *zenſezenſe*, and doth ſignifie the number following the ſame, to be the product of the third multiplication, that is the product of the ⅋ foure times taken and multiplied. The fift is called *ſurſolide*, and doth ſignifie the number following the ſame, to be the product of the fourth multiplication. The ſixt is called *zenſecube*, and doth ſignifie the number following the ſame, to be the product of the fift multiplication. The ſeuenth is called *bſurſolide*, or ſecond ſurſolide, and doeth ſignifie the number following the ſame, to be the product of the ſixt multiplication. The eight is called *zenſezenſezenſe*, & doth ſignifie the number following the ſame, to be the product of the ſeuenth multiplication. The ninth is called *cubecube*, and doth ſignifie, the number following the ſame, to be the product of the eight multiplication. The tenth is called *zenſeſurſolide*, & doth ſignifie the number following the ſame, to be the product &c. The eleuenth, is called *cſurſolide*, or third ſurſolide. The twelfth is called *zenſezenſecube*. The thirteenth is called *dſurſolide*, or fourth ſurſolide. You may proceede further at your pleaſure, if you marke that the fift caracter is the firſt ſurſolide, the ſeuenth the ſecond ſurſolide, and the next vncompound number following, the next ſurſolide, & ſo infinitely euer the next vncompound

pound number following the next ſurſolide: then alſo that deuiding the number of any caracter by the leaſt number that can meaſure it, and that quotient, by the leaſt number that can meaſure it, and againe that quotient by the leaſt number that can meaſure it, and ſo euer deuiding the laſt quotient by the leaſt number that can meaſure it, vntill the quotient be an vnit, that the deuiders ſhew which caracters ioyned together, make the caracter demaunded.

Examples.

If it be demaunded what caracter the 42 caracter is, firſt deuide 42 by 2, the quotient is 21, which deuide by 3, the quotient is 7, which deuide by 7, the quotient is 1: now the deuiders 2 3 and 7 do ſhew, that the ſecond third, and ſeuenth caracters ioyned together thus Ⳑ ᘓ b ſ 8, is the caracter deſired, whoſe pronunciation is *zenſecubebſurſolide*, and ſignifieth the number following the ſame, to be the product of the 41 multiplication, or the product of the ᙆ taken 42 times & multiplied. The 684 caracter is Ⳑ Ⳑ ᘓ ᘓ f ſ 8, and is pronounced *zenſezenſecubecubeffurſolide*, and ſignifieth the number following the ſame, to be the product of the 683 multiplication, &c. infinitely.

The ſecond inſtruction is of the extractions of rootes.

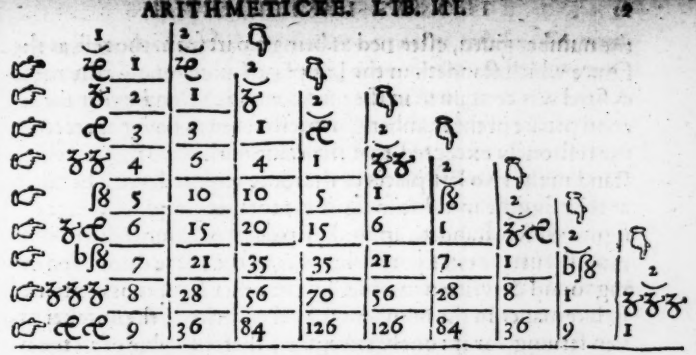
ANy number giuen, being the Ⳑ, the ᘓ, the ⳐⳐ, the ſ 8, or any other product of a ᙆ many times taken and multiplied, hath his root to be found in this maner following. Make a pricke ouer the figure that ſtandeth in the firſt place of the number giuen, and other prickes ouer the figures following, euer leauing ſo many figures vnpricked betweene each two prickes, as the number giuen is ſayd to be the product of multiplicatiõs. And looke how many prickes are made in this order ouer the number giuen, and with ſo many figures the value of his roote is ſignified. Then abate the greateſt number you can being the product of ſo many multiplications as the number giuen is ſayd to be the product of multiplications, from the number which ſtãdeth vnder the firſt prick toward the left hand of the number

giuen, eſteemed in value as if it were apart frō the reſt (euer reckening the figure or figures which ſtand at the left hand of any pricke, to ſtand alſo vnder it) and write the remainer ouer it, ſtriking out as you do in deuiſion. The roote of this number abated, is the firſt figure found of the ☉ required, and ſtandeth in the laſt place of it. Now know that of ☽ numbers, the next figure of his root being foūd and written by this toward the right hand, is contained

This Table muſt bee knowne by heart, for it doth ſhew the ☽. ☌. ☽☽. and ſ8. of any number not aboue nine, and may be prolonged at your pleaſure.

☉.	☽ .	☌ .	☽☽ .	ſ8
9 .	81 .	729 .	6561 .	59049
8 .	64 .	512 .	4096 .	32768
7 .	49 .	343 .	2401 .	16807
6 .	36 .	216 .	1296 .	7776
5 .	25 .	125 .	625 .	3125
4 .	16 .	64 .	256 .	1024
3 .	9 .	27 .	81 .	243
2 .	4 .	8 .	16 .	32
1 .	1 .	1 .	1 .	1

in the number which is, or is come to be vnder the next pricke of the number giuen, taken as if it were apart from the reſt, two times the value of the firſt figure found of the roote, and once it ſelfe times, euer eſteeming each figure of the roote in value as it ſignifieth ſtanding ſo together as is aforeſaid. Of ☌ numbers, it is contained three times the ☽ of the firſt figure found of the roote, and three times the product of the ſayd firſt figure found multiplied by it, and once the ☽ times of it ſelfe. Of ☽☽ numbers it is contained foure times the ☌ of the firſt figure found of the roote, and ſixe times the product of the ☽ of the ſaid firſt figure multiplied by it, and foure times the product of the ſayd firſt figure multiplied by the ☽ of it, and once the ☌ times of it ſelfe. Of ſ8 numbers it is contained fiue times the ☽☽ of the firſt figure found of the roote, and ten times the product of the ☌ of the ſaid firſt figure multiplied by it, and ten times the product of the ☽ of the ſayd firſt figure multiplied by the ☽ of it, & fiue times the product of the ſayd firſt figure multiplied by the ☌ of it, and once the ☽☽ times of it ſelfe. The like conſideration you muſt haue if the number giuen be the product of anie other multiplication, which for your eaſe this Table following will bring to your remembrance, & is to be augmented at your pleaſure.



1	1	1	1	1	1	1	1	1	1
2	1	2	3	4	5	6	7	8	9
3	3	6	10	15	21	28	36		
4	6	10	20	35	56	84			
5	10	15	35	70	126				
6	15	21	56	126					
7	21	28	84						
8	28	36							
9	36								

pleaſure. The ſecond rowe of numbers downward is made of the firſt by addition, the third of the ſecond, the fourth of the third,&c. infinitely. Euer agreeing to the product of any roote ſo many times taken and multiplied. The caracters on the left hand are any number giuen, and the numbers in that line toward the right hand, are the numbers ſeruing to extract his ſuch root, in this maner. As if the nũber giuẽ be ſaid to be a ccc nũber: it ſheweth that if the figure which ſtandeth in the laſt place of his roote, haue the next figure of his root written by it, as we haue before declared, the ſayd next figure to be contained in the nũber which is, or ſhal come to be, vnder the ſecond prick of the number giuẽ eſteemed as being apart from the reſt, 9 times the ȝȝȝ of the firſt figure foũd of his root, and 36 times the product of the bſ of the ſayd firſt multiplied by it, & 84 times the product of the ȝcc of the ſaid firſt multiplied by the ȝ of it, & 126 times the product of the ſ of the ſaid firſt multiplied by the cc of it, & 126 times the product of the ȝȝ of the ſaid firſt multiplied by the ȝȝ of it, & 84 times the product of the cc of the ſaid firſt multiplied by the ſ of it, & 36 times the product of the ȝ of the ſaid firſt multiplied by the ȝcc of it, & 9 times the product of the ſayd firſt multiplied by the bſ of it, & once the ȝȝȝ times of it ſelf. And thus vnderſtãd of all other. The next figure of any roote required being found, and written in order by the other two figures, toward the right hand, is contained in the nũber which is or ſhal come to be vnder the third prick of

the number giuen, eſteemed as being a part from the reſt, as the figure which ſtandeth in the laſt place ſauing one of the roote deſired was contained in the number which ſtood vnder the ſecond pricke of the number giuen, eſteemed as being apart from the reſt, onely excepted, that the value of the two figures which ſtand in the two laſt places of the roote required, muſt be taken as they ſignifie in value ſo written together, in place where the figure which ſtandeth in the laſt place was before taken alone. And ſo euery next figure following of the roote demanded being found & written in order by the other figures, is contained in like maner in the number vnder the pricke of the number giuen ſeruing for it: onely excepted, that the value of all the figures toward the left hand of it, muſt be taken together in place where the figure which ſtandeth in the laſt place was before taken alone. Therefore of ʒ numbers, deuide the number which is or is come to be vnder the ſecond pricke of the number giuen (eſteemed as being apart from the reſt) by the product of the firſt figure found of the roote (eſteemed in value as it ſignifieth when the next figure is written by it) multiplied by 2, hauing regard that the ʒ of the quotient may be abated out of the remainer, and then that ſuch quotient is the next figure of the roote required. Of ☌ numbers, deuide it by the product of the ʒ of the firſt figure found of the roote multiplied by 3, hauing regard that the product of the product of the firſt figure found of the root multiplied by 3 multiplied by the ʒ of the quotiēt, may be abated out of the remainer, and yet that the ☌ of the quotient may be abated out of that laſt remainer, and then that ſuch quotient is the next figure of the roote required. Of ʒʒ numbers deuide it by the product of the ☌ of the firſt figure found of the roote multiplied by 4, hauing regard that the product of the product of the ʒ of the firſt figure found of the root multiplied by the ʒ of the quotient multiplied by 6 may be abated out of the remainer, and that out of the remainer of this remainer may be abated the product of the product of the firſt figure found of the roote multiplied by the ☌ of the quotient multiplied by 4, and yet that the ʒʒ of the quotient may be abated out of the remainer of that remainer, and then that ſuch

quotient

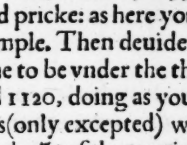
quotient is the next figure of the roote required. In like maner (as is aforesayd) you may find all the figures that signifie the value of the roote of any number giuen, &c.

Examples.

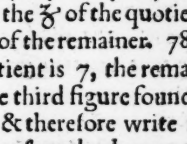
If this ȝ number 321489 be giuen to haue his ȝ roote extracted: first pricke the ȝ number giuen (according to this instruction) leauing one figure vnpricked betweene each two prickes: as here you see it done in the example. Then considering that 25 is the greatest ȝ number that can be abated out of 32, the number vnder the first pricke, therefore abate 25 from it, and write the remainer ouer the same, striking out as you do in deuision, and write the roote ȝ of the sayd 25, which is 5, in the quotient: as here you see it done in the example. Then multiply the aforesayd 5 (the first figure found of the roote demaunded) esteemed in value as it signifieth when the next figure of the roote is written by it (is 50) by 2, the product is 100, this 100 write vnder the 714, the number which is come to be vnder the second pricke: as here you see it done in the example. Then deuide 714 (which is come to be vnder the second pricke) by the aforesaid 100, doing as you do in other deuisions (onely excepted) with hauing regard that the ȝ of the quotient may be abated out of the remainer. 714 so deuided the quotiēt is 6 and the remainer is 114, which sayd 6 is the second figure found of the roote required, & therfore write it in the quotiēt by the 5 there first foūd, as here you see it done in the example. Then abate 36 (the ȝ of the sayd 6) out of 114 (the remainer there) striking out as you do in deuision, & writing the remainer ouer it: as here you see it done in the example. Then multiply 56 (the two first figures found of the roote demanded) esteemed in value as they signifie when the next figure of

```
  .  .  .
321489 (
25
  .  .  .
7 .  .
321489 (5
100
  .  .  .
7 .  .
321489 (56
100
```

the root is written by thē (is 560) by 2, the product is 1120, this 1120 write vnder 7889, the nūber which is come to be vnder the third pricke: as here you ſee it done in the example. Then deuide 7889 (which is come to be vnder the third pricke) by the ſayd 1120, doing as you do in other deuiſions (only excepted) with hauing regard that the ȝ of the quotient may be abated out of the remainer. 7889 ſo deuided the quotient is 7, the remainer 49, which 7 is the third figure found of the roote deſired, & therefore write it by the 56 there before found; as here you ſee it done in the exāple. Then abate the ȝ of the ſayd 7, which is 49, out of the remainer there, which is 49, ſtriking out as you do in deuiſion, and as you ſee it here done in the example. Thus in this maner the extraction of the ȝ root of that ȝ number giuen is finiſhed, and his roote ȝ is found to be 5 6 7 exactly, & nothing remaining: as here you ſee in the example. And in like maner you may extract the ȝ roote of any other ȝ number giuen.



If this ☌ number 18228 4263 be giuen, to haue his ☌ roote extracted. Firſt pricke the ☌ number giuen (according to this inſtruction) leauing two figures vnpricked betweene each two prickes, as here you ſee it done in the example. Then conſidering that 125 is the greateſt ☌ number, that can be abated out of 182 the number vnder the firſt pricke, therefore abate the ſame from it, ſtriking out as you do in deuiſion, writing the remainer ouer the ſame. And write the roote ☌ of the ſayd 125, which is 5, in the quotient: as here you ſee it done in the example. Then the ſaid 5 being the firſt figure foūd of the roote demaunded, eſteemed in value as it ſignifieth when the next figure of the roote is written by it, is 50, his ȝ is 2500, this 2500 multiply by 3, the product is 7500, by this



this 7500 deuide 57284 (the number which is come to be vnder the secõd pricke) but hauing regard that the product of the product of the first figure found of the root (esteemed as is aforesayd) multiplied by 3 multiplied by the ℨ of the quotient, may be abated out of the remainer: and yet that the ℭ of the quotient may be abated out of that last re-remainer. 57284 so deuided the quotient is 6, which 6 is the second figure found of the roote required, therefore write it in the quotient by the 5 there first found: as here you see it in the example. Then multiply the deuider 7500 by 6 (the last quotient) the product is 45000. &

```
                  57
        x8228426 3(56
   ℭ50 . . ℨ2500
                  3
        ---------
        7500 the deuider
```

multiply 50 (the first figure found of the root, esteemed in value as it signifieth when the next figure of the root is written by the same) by 3, the product is 150, this 150 multiply by 36 (the ℨ of the last quotiēt) the product is 5400. and the ℭ of the last quotient is 216. Thē adde the aforesaid 45000, 5400 and 216 together. Their aggregat is 50616, which aggregat abate from the number which is come to be vnder the second pricke, striking out and writing the remainer ouer it: as here you see it done in the exāple. Then 56 the two first figures foūd of the root demanded, esteemed in value as they signifie when the next figure of the roote is written by them, is 560, his ℨ is 313600, this 313600 multiply by 3, the product is 940800, by this product 940800 deuide 6668263 (the number which is come to be vnder the third pricke) but hauing regard that the product of the

```
                  6
        x8228426 3(56
   ℭ50 ℨ2500
   1 . . 3 . 3
         150   7500
   ℭ216 ℨ36   ℭ6
         5400  45000
               5400
                216
               50616
```

```
                  57668
        x8228426 3(567
   ℭ560 . . ℨ313600
                  3
        ---------
        940800 the deuider.
```

product of the two firſt figures found of the roote (eſteemed as is aforeſayd) multiplied by 3 multiplied by the ȝ of the quotient may be abated out of the remainer, and yet that the ȼ of the quotient may be abated out of that laſt remainer. 6668263 aforeſayd ſo deuided, the quotient is 7, which 7 is the third figure found of the roote required, therefore write it in the quotient by the 56 there before found: as here you ſee it done in the example. Thē multiply the deuider 940800 by 7 (the laſt quotient) the ꝓduct is 6585600. And multiplie 560 (the figures before foūd of the roote required, eſteemed in value as they ſignifie when the next figure of the roote is written by them) by 3 the product is 1680. This 1680 multiply by 49 (the ȝ of the laſt quotient) the product is 82320, and the ȼ of the laſt quotient is 343. Then adde the aforeſayd 6585600, 82320, and 343 together, their aggregat is 6668263, which aggregat abate frō the number which is come to be vnder the third pricke, ſtriking out &c. as here you ſee it done in the example. Thus in this maner the extraction of the ȼ roote of that ȼ number giuen is finiſhed: and his roote ȼ is found to be 567 exactly, and nothing remaining: as here you ſee in the example. And in like maner you may extract the ȼ roote of any other ȼ number giuen.

```
                    6
                  57668
          18228426ȝ (567
  ȼ560 . . ȝ313600
1 . . 3 . .      3
    1680      940800
ȼ343 . . ȝ49 . . ȼ7
    82320     6585600
               82320
                 343
              6668263
```

If this ȝȝ number 10335517721, be giuen to haue his ȝȝ roote extracted: firſt pricke the ȝȝ number giuen (according to this inſtruction) leauing three figures vnpricked betweene each two prickes: as here you ſee it done in the example. Then conſidering that 625 is the greateſt ȝȝ number that can be abated out of 1033, the number vnder the firſt pricke: therfore

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10335517721 (
```

abate

abate the same from it striking out and writing the remainer as you do in deuision, and the roote ȝȝ of the sayd 625, which is 5 write in the quotient: as here you see it done in the example. Then the sayd 5, which is the first figure found of the roote required, esteemed in value as it signifieth when the next figure of the roote is written by it, is 50, his ȝ is 2500, his cc is 125000, this cc 125000 multiply by 4 the product is 500000,

```
                 408
 4085517 (5      ---
 ---------       4085517 (56
 ȝȝ 25 . . ȝ 2500 . . cc 125000
                          4
                 ---------
                 500000 the deuider
```

by this 500000 deuide 4085517, the number which is come to be vnder the second prieke: but hauing regard that the product of the product of the ȝ of the first figure found of the roote esteemed in value as is aforesaid, multiplied by 6 multiplied by the ȝ of the quotient may be abated out of the remainer. And also that the product of the product of the first figure found of the roote esteemed in value as is aforesaid multiplied by 4 multiplied by the cc of the quotient may be abated out of that remainer then remaining, and yet that the ȝȝ of the quotient may be abated out of that last remainer. 4085517 aforesayd so deuided the quotient is 6, which 6 is the second figure found of the roote desired, therefore write it in the quotiēt by the 5 there first foūd: as here you see it done in the example. Thē multiply the deuider 500000 by 6 the last quotient, the product is 3000000. And mul-

```
                  50
 4081021          --
 ---------        4081021 (56
 ȝȝ 1296 . . cc 216 . . ȝ 36 . . 6
 1 . . . 4 . . 6 . . 4
 200       15000     500000
 43200     540000    3000000
                     540000
                      43200
                       1296
                    -------
                    3584496
```

tiply the ʒ of the firſt figure found of the roote eſteemed in value as is aforeſaid, which is 2500 by 6 the product is 15000, this 15000 multiply by the ʒ of the quotiēt, the prodnct is 540000. And multiply the firſt figure found of the roote eſteemed as is aforeſayd, which is 50 by 4, the product is 200, this 200 multiply by 216 the ☌ of the quotient, the product is 43200, and the ʒʒ of the quotient is 1296. Then adde the ſayde 3000000, 540000, 43200 and 1296 together, their aggregate is 3584496, which aggregat abate from 4085517 the number which is come to be vnder the ſecond pricke, ſtriking out and writing the remainer, as here you ſee it done in the example Thē 56 the two firſt figures found of the roote required, eſteemed in value as they ſignifie when the next figure of the roote is written by them, is 560, his ʒ is 313600, his ☌ is 175616000, this ☌ 175616000 multiply by 4 the product is 702464000, and by this 702464000 deuide 501021712 1 the number which is come to be vn-

50

4081021

501021712 1 (567

☌560 . . ʒ313600 . . ☌175616000

4

702464000 *deuider.*

der the third pricke: but hauing regard that the product of the product of the ʒ of the two firſt figures found of the root eſteemed as is aforeſaid, multiplied by 6 multiplied by the ʒ of the quotient, may be abated out of the remainer, and alſo that the product of the product of the two firſt figures found of the roote eſteemed as is aforeſayd multiplied by 4 multiplied by the ☌ of the quotient may be abated out of that remainer then remaining, and yet that the ʒʒ of the quotient may be abated out of that laſt remainer. 501021712 1 aforeſayd ſo deuided the quotient is 7, which 7 is the third and laſt figure found of the roote required. Therefore write it in the quotient by the 56 there before found, as here you ſee it done in the example. Then multiply the deuider 70246400 by 7 (the quotient) the pro-

50
4081021
2033552771 21 (567

ȝ560 . . ȝ313600 . . ȝ175616000
1 . . 4 . . 6 . . 4
2240 1881600 702464000
ȝȝ2401 . . ȝ243 . . ȝ49 . . ȝ7
768320 . . 92198400 . . 4917248000
92198400
768320
2401
5010217121

product is 4917248000, and multiply the ȝ of the two firſt figures found of the roote (eſteemed as is aforeſayd) which is 313600 by 6, the product is 1881600, this 1881600 multiply by 49 the ȝ of the quotient, the product is 92198400. And multiply the two firſt figures found of the roote (eſteemed as is aforeſayd) which is 560 by 4 the product is 2240, this 2240 multiply by 343 the ȝ of the quotient, the product is 768320 and the ȝȝ of the quotient is 2401. Then adde the aforeſayd 4917248000, 92198400, 768320 & 2401 together, their aggregat is 5010217121, which aggregat abate from the number which is come to be vnder the third pricke, ſtriking out &c. as you ſee here done in the example. Thus in this maner the extraction of the ȝȝ roote of that ȝȝ number giuen is finiſhed, and his roote ȝȝ is found to be 567 exactly, and nothing remaining: as here you ſee in the example. And in like maner you may extract the ȝȝ root of any other ȝȝ number giuen.

If this ſ number 83084095520768 be giuen to haue his ſ roote extracted; firſt pricke the ſ number giuen (according to this inſtruction) leauing foure figures vnpricked betweene each two prickes: as here you ſee it done in the example. Then cõ- 8308̇4095̇5207̇68̇ (ſidering that 7776 is the greateſt ſ number that can be abated out of 8308, the number which is

vnder the firſt pricke, therefore abate the ſame from it ſtriking out and writing the remainer as you do in deuiſion: and the root ſg of the ſayd 7776 which is 6 write in the quotient: as here

532
8ʒ08409552076ȧ(6

you ſee it done in the example. Then the ſayd 6 (which is the firſt figure found of the roote deſired) eſteemed in value as it ſignifieth when the next figure of the roote is written by it: is 60, his ʒ is 3600, his ☌ is 216000, his ʒʒ is 12960000, this 12960000 multiply by 5, the product is 64800000

532
8ʒ08409552076ȧ(60

☌ 60 . . ʒ 3600 . . ☌ 216000 . . ʒʒ 12960000

5

64800000 *deuider.*

by this 64800000 deuide 532409552 the number which is come to be vnder the ſecond pricke, but hauing regard that the product of the product of the ☌ of the firſt figure found of the roote eſteemed in value as is aforeſayd, multiplied by 10 multiplied by the ʒ of the quotient may be abated out of the remainer: and alſo that the product of the product of the ʒ of the firſt figure found of the roote eſteemed in value as is aforeſayd, multiplied by 10 multiplied by the ☌ of the quotient may be abated out of that remainer then remaining: and againe, that the product of the product of the firſt figure found of the roote eſteemed in value as is aforeſayd, multiplied by 5 multiplied by the ʒʒ of the quotient may be abated out of that remainer thē remaining: and yet that the ſg of the quotient may be abated out of that laſt remainer. 532409552 aforeſayd ſo deuided the quotient is 0, which 0 is the ſecond figure found of the root required, therefore write it in the quotient by the 6 there firſt found: as here you ſee it done in the example. And ſo the worke which appertaineth thus farre for the extraction of the roote required is finiſhed, becauſe the quotient is 0, which 0 doth produce nothing howſoeuer multiplied. Then 60 the two firſt figure

figures found of the roote desired esteemed in value as they signifie when the next figure of the roote is written by them, is 600, his ☽ is 360000, his ☌ is 216000000, his ☽☽ is 129600000000, this 129600000000 multiply by 5, the product is 648000000000, by this 648000000000 deuide 53240955207 68 the number which is come to be

532
8ø384095520768(608

☌ 600 .. ☽ 360000 .. ☌ 216000000 .. ☽☽ 129600000000
5
deuider 648000000000

vnder the third pricke, but hauing regard that the product of the product of the ☌ of the two first figures found of the roote esteemed in value as is aforesayd, multiplied by 10 multiplied by the ☽ of the quotiēt may be abated out of the remainer. And also that the product of the product of the ☽ of the two first figures found of the roote esteemed in value as is aforesayd, multiplied by 10 multiplied by the ☌ of the quotient may be abated out of the remainer then remaining: and againe, that the product of the product of the two first figures found of the root esteemed in value as is aforesayd, multiplied by 5 multiplied by the ☽☽ of the quotient may be abated out of the remainer then remaining: and yet that the ☾ of the quotient may be abated out of that last remainer. 53240955207 68 aforesayd so deuided the quotient is 8, which 8 is the third and last figure found of the roote required, therefore write it in the quotient by the 60 there before found: as here you see it done in the example. Then multiply the deuider 648000000000 aforesayd, by 8 the last quotient, the product is 5184000000000, and multiply the ☌ of the two first figures found of the roote esteemed in value as is aforesayd, which is 216000000 by 10, the product is 2160000000, this 2160000000 multiply by 64 the ☽ of the quotient, the product is 138240000000. And multiply the ☽ of the two first figures found of the roote esteemed in value as is aforesayd, which is 360000 by 10, the

product is 3600000, this 3600000 multiply by 512 the ce of the quotient, the product is 1843200000. And multiply the two first figurs found of the roote esteemed in value as is aforesayd, which is 600 by 5, the product is 3000, this 3000 multiply by 4096 the ȝȝ of the quotient, the product is 12288000, and the ſſ of the quotient is 32768. Then adde the aforesaid 5184000000000, 138240000000, 1843200000, 12288000 and 32768 together, their aggregate is 532409-5520768, which aggregate abate from the number which is come to be vnder the third pricke, striking out &c. as you see it done in the example. Thus in this maner the extraction of the ſſ roote of that ſſ number giuen is finished, and his ſſ roote is found to be 608 exactly, and nothing remaining: as here you see in the example. And in like maner you may extract the ſſ roote of any other ſſ number giuen, &c.

Abreuiation of the former workes.

THe abreuiation of Arithmeticall works, doth consist as well in the inuention howe to omit vnnecessarie figures and iterate placings of numbers, as to find the neareſt way to attaine

taine vnto the purpoſe pretended. Yet becauſe all men be not of like quickneſſe of capacitie, neither of like greatneſſe of inuention, we will ſet downe ſome briefer operations to worke the aforeſayd extractions of rootes then the aforegiuen, and ſo leaue what more may be ſayd in this matter to each mans reach of wit and liking.

Examples.

If this aforeſayd ℨ number 321489 be giuen to haue his roote ℨ extracted: pricke the number giuen, and find the laſt figure of the roote in the ſame maner that you do in the former operation: as here you ſee it done in the exãple.

7
3̇2̇1̇4̇8̇9̇(5

Then multiplie 5 (the firſt figure found of the roote) by 2 the product is 10, this 10 write vnder 714 (the number which is come to be vnder the ſecond pricke) and vnder the ſecond place of the ſame (for the cauſe aforeſayd in the former worke: and is, that the ſayd 5 muſt be eſteemed in value as it ſignifieth when the next figure of the root is written by it) as here you ſee it done in the example. Then deuide 71 (the number which is written ouer the 10 ſo laſt written) by the ſayd 10 hauing regard (as is aforeſayd in the former worke) that the ℨ of the quotient may be abated out of the remainer of 714 (the number which is come to be vnder the ſecond pricke) 71 ſo deuided the quotient is 6, which 6 is the ſecond figure found of the roote required, therefore write and worke as here you ſee it done in the example. Then abate 36 (the ℨ of the quotient) from 114 (the remainer) ſtriking out, and writing, as here you ſee it done in the example. Then multiply 56 (the two firſt figures found of the root) by 2 the product is 112. This 112 write vnder 7889 (the number which is come to be vnder the third pricke) and vnder

7
3̇2̇1̇4̇8̇9̇(56
10

1
7
3̇2̇1̇4̇8̇9̇(56
10

2
778
3̇2̇1̇4̇8̇9̇(56
10

the ſecond place of the ſame (for the cauſe aforeſayd in the former worke, and is that the 56 muſt be eſteemed in value as they ſignifie whē the next figure of the root is written by them) as here you ſee it done in the exāple.

```
  x
  778
3xx489(56
 x012
    1
```

Then deuide 788 which ſtandeth ouer 112 ſo laſt written, by the ſayd 112: hauing regard (as is aforeſayd) that the 3 of the quotient may be abated out of the remainer of 7889 (the number which is come to be vnder the third pricke) 788 ſo deuided the quotient is 7, which 7 is the third figure found of the roote required. Therfore write and worke as here you ſee it done in the example. Then abate 49 (the 3 of the quotient) from 49 (the remainer) ſtriking out &c. as here you ſee it done in the example. Thus in this maner the roote 3 of the 3 number giuen is extracted, with ſome more breuitie then by the former worke, as is apparent by the examples: and in like maner, and many other wayes, you may extract the roote 3 of any other 3 number giuen.

```
  x
  7784
3xx489(567
 x0xx
    x
```

```
  x
  7784
3xx489(567
 x0xx
    x
```

If this aforeſaid cc number 18228426 3 be giuen to haue his roote cc extracted. Pricke the number giuen, and find the laſt figure of the root, in the ſame maner that you do in the former operation: as here you ſee it done in the example. Then multiply 5 the firſt figure found of the roote by 3 the product is 15, this 15 write vnder 57284 (the number which is come to be vnder the ſecond prick) and vnder the ſecond place of the ſame: becauſe the ſayd 5 muſt be eſteemed in value as it ſignifieth whē the next figure of the root is written by it. Then multiply the ſayd 15 by

```
   57
x8228426 3(5
```

```
   57
x8228426 3(5
   15
```

```
   75
```

5 (the

5 (the first figure found of the roote) the product is 75, this 75 write vnder the said 15 last written, and vnder the second place of the same, because the sayd 5 must be esteemed in value as it signifieth when the next figure of the roote is writtē by it. Then by this sayd 75 deuide 572 (the number written ouer the same) hauing regard that the product of the ℨ of the quotient multiplied by the last sayd 15 may be abated out of the remainer of 5728 (the number ouer the said 15) and yet that the ☌ of the quotient may be abated out of the remainer of 57284 (the number which is come to be vnder the second pricke) 572 aforesayd so deuided the quotient is 6, which 6 is the second figure found of the roote desired; therefore write it in the quotient by the 5 there afore found, as here you see it done in the example. Then draw a line vnder the sayd deuider 75, and multiply it by 6 (the quotient) the product is 450, writing the same vnder it vnder the line. And multiply the last sayd 15 by 36 (the ℨ of the quotient) the product is 540, writing it vnder the same vnder the line. And the ☌ of the quotient is 216 writing it vnder the line vnder the number which is come to be vnder the second pricke. Then adde those numbers so written vnder the line together, their aggregat is 50616, which abate from 57284 (the number which is come to be vnder the second pricke) striking out and writing all things as here you see it done in the example. Then multiply 56 the two first figures found of the roote by 3 the product is 168, this 168 write vnder 66682 63 (the number which is come to be vnder the third prick) and (for the cause aforesayd) vnder the second place of the same. Then multiply the sayd 168 by 56 (the two first figures found of the roote) the product is 9408, this 9408 write vnder the sayd 168 last written, and (for the cause aforesayd) vnder the second place of the same. By this sayd 9408 deuide 66682 the number written ouer the same: hauing regard that the product of the ℨ of the quotient multiplied by the last

```
        6
    57668
x82284263(56
      15
     75
    ----
     450
      540
       216
    -----
    50616
```

ſayd 168 may be abated out of the remainer of 666826 (the number ouer the ſayd 168) and yet that the ☞ of the quotient may be abated out of the remainer of 6668263 (the number which is come to be vnder the third pricke) 66682 aforeſayd ſo deuided the quotiēt is 7, which 7 is the third figure found of the roote required. Therfore write it in the quotient by the 56 there before found. Then drawe a line vnder the ſayd deuider 9408 and multiply it by 7 (the quotient) the product is 65856, writing the ſame vnder it vnder the line: and multiply the laſt ſayd 168 by 49 (the ☞ of the quotient) the product is 8232 writing it vnder the ſame vnder the line: and the ☞ of the quotient is 343, writing it vnder the line vnder the number which is come to be vnder the third pricke. Then adde thoſe ſayd numbers (ſo written vnder the line) together; their aggregat is 6668263, which 6668263 abate from 6668263 the number which is come to be vnder the third pricke, ſtriking out and writing all things as here you ſee it done in the example. Thus in this maner the ☞ roote of the ☞ nūber giuen is extracted, with ſome more breuitie then by the former worke. And in like maner you may extract the roote ☞ of any other ☞ number giuen. Yet for your more inſtruction conſider this third operation following.

```
        6
      57668
x82284263(56
       15
      75
     450
      540
       216
     50616
        168
     9408

        6
      57668
x82284263(567
       15
      75
     450
      540
       216
     50616
        168
     9408
     65856
       8232
         343
     6668263
```

If this aforeſayd ☞ number 182284263 be giuen to haue

his roote cꝭ extracted: pricke the number giuen, and find the laſt figure of the roote, in the ſame maner that you do in the two former operations: as here you ſee it done in the example. Then multiply 5 (the firſt figure found of the root) by 3, the product is 15, this 15 write vnder 57284 and vnder the ſecond place of the ſame. Then multiply the ſayd 15 by 5 (the firſt figure found of the root) the product is 75, this 75 write vnder the ſayd 57284, and vnder the third place of the ſame. Then deuide 572 the number ouer the ſayd 75 by the ſame 75, hauing regard &c. as is ſayd in the two former operations: the quotient is 6, which 6 write in the quotient by the 5 there found, multiplying the deuider by it, and abating the product from 572, as in deuiſion. Then multiply the aforeſayd 15 by 36 (the 3 of the quotient) abating the product from 1228, as in deuiſion. Then the cꝭ of the quotient is 216, which abate from 6884, ſtriking out and writing all things as here you ſee it done in the examples. Then multiply 56 (the two firſt figures found of the roote) by 3, the product is 168, this 168 write vnder 6668263, and vnder the ſecond place of the ſame. Then multiply the ſaid 168 by 56 (the two firſt figures found of the roote) the product is 9408, this 9408 write vnder the ſayd 66684263, and vnder the third place of the ſame. Then deuide 66682 the number ouer the ſayde 9408 by the ſame 9408, hauing

57
x8228426̇3̇(5

57
x8228426̇3̇(5
15

57
x8228426̇3̇(5
715
5

12
57
x8228426̇3̇(56
715
5

6
22
578
x8228426̇3̇(56
715
5

6
226
57868
x8228426̇3̇(56
715168
5

regard &c. as is ſayd in the former worke, the quotient is 7, which 7 write in the quotiēt by the 56 there found, multiplying the ſayd deuider by it, and abating the product from 66682 as in deuiſion. Then multiply the ſayd 168 by 49 (the ☉ of the quotient) abating the product from 8266 as in deuiſion. Then the ☌ of the quotient is 343, which abate from 343, ſtriking out & writing al things as here you ſee it done in the examples. Thus in this maner the roote ☌ is extracted with ſome more breuitie, then by the two former operations, and in like manner, and many other wayes you may extract the roote ☌ of any other ☌ number giuen.

6
x26
57868
x8228426 3 (56
7x5168
5408
9

If this aforeſayd ☉☉ number 1033551271121 be giuen to haue his roote ☉☉ extracted, firſt extract the roote ☉ of the number giuen: and then extract the roote ☉ of that roote: for the roote ☉ of that roote is the roote ☉☉ of the number giuen. Thus in this maner the roote ☉☉ of the ☉☉ number is extracted with ſome more breuity then in the former operation, as you may ſee it done in the example on the next page. And in like maner, and many other wayes,

6
x26
57868
x8228426 3 (56
7x5168
5408
9

6
x26823
5786864.
x8228426 3 (567
7x5168
5408
9

6
x26823
5786864.
x8228426 3 (567
7x5168
5408
9

you may extract the root ☉☉ of any other ☉☉ number giuen.

If

5

35778

x 9x42x67 . 778 .

x033551771212(32x489(567

66442z896 x0x2

66442 x

6

If this aforesayd ſſ number 830840955 20768 be giuē to haue his roote ſſ extracted: pricke the number giuen, and find his roote in the same maner that you do in the former operation. But omit the nothings here omitted, which are written in the former worke. Yet hauing regard, to value, place & write all numbers as if they had the sayd nothings omitted written by them: as here you see it done in the example following, and therefore needeth no further declaration.

532

830840955 20768(608

ȝȝ1296 . ȼ216 . ȝ36 . ȥ6

5

deuider 6480

ȝȝ1296 . ȼ216 . ȝ36 . ȥ6

5 . 10 . 10 . 5 . 1

deuider 6480 . 2160 . 360 . 30

ȥ8 . ȝ64 . ȼ512 . ȝȝ4096 . ſſ32768

51840 . 138240 . 184320 . 122880

138240

184320

122880

32768

532409552 0768

Thus in this maner the root ſſ of the ſſ number giuen is extracted with some more breuitie then in the former operation, and in like maner you may extract the roote ſſ of any other ſſ number giuen. Yet for your more instructiō consider this third operation following.

If this aforeſaid ſ8 number 8308409552076 8 be giuen to haue his roote ſ8 extracted, pricke the number giuen, and find the laſt figure of the roote in the ſame maner that you do in the two former operations: as here you ſee it done in the example.

532
8308409552076 8(6

Then multiply 1296 (the ȝȝ of the quotient) by 5 the product is 6480, this 6480 write vnder 53240955 (the number which is come to be vnder the ſecond pricke) and vnder the fift place of the ſame, ſo that omitting 0, vnder 532 is written 648. Then deuide 532 the number ouer 648 by the ſame 648, hauing regard &c. as is ſaid in the former worke. The quotient is 0, which 0 write in the quotiēt by the 6 there found, multiplying the deuider by it, the product is nothing. And ſo the worke which appertaineth thus farre for the extraction of the roote is finiſhed: ſtriking out and writing all things as here you ſee it done in the example. Thē multiply 12960000 the ȝȝ of 60 (the two firſt figures found of the roote) by 5, the product is 648000000, this 648000000 write vnder the number which is come to be vnder the third prieke: and vnder the fift place of the ſame. So that 00000 being omitted 648 is written vnder 5324. By which 648 deuide 5324, hauing regard &c. as is ſayd in the former operation. The quotient is 8, which 8 write in the quotient by the 60 there foūd: multiplying the deuider by it, and abating the product from the number ouer the ſame as in deuiſion. Then write the product of 2160000 (the ☌ of the 2 firſt figures found

532
8308409552076 8(60
648

532
8308409552076 8(60
6488
64

532
8308409552076 8(60
6488
14
53200

532
8308409552076 8(608
648816
642

found of the roote) multiplied by 10, vnder the number which is come to be vnder the third pricke, & vnder the fourth place of the ſame. So that 0000 being omitted, 216 is written vnder 14009, from which abate the product of the ſayd 216 multiplied by 64 (the ℨ of the quotient) as in deuiſion. Then write the product of 3600 (the ℨ of the two firſt figures found of the root) multiplied by 10: vnder the nũber which is come to be vnder the third pricke, and vnder the third place of the ſame. So that 000 being omitted, 36 is written vnder 18555: from which abate the product of the ſayd 36 multiplied by 512 (the ☌ of the quotient) as in deuiſion. Thẽ write the product of 60 (the two firſt figures found of the roote) multiplied by 5 vnder the number which is come to be vnder the third pricke, and vnder the ſecond place of the ſame. So that 00 being omitted, 3 is written vnder 12320. From this 12320 abate the product of the ſayd 3 multiplied by 4096 (the ℨℨ of the quotient) as in deuiſion. Then the ſ8 of the quotient is 32768, which abate from 32768 (the laſt remainer which is come to bee vnder the third pricke) ſtriking out and writing all things as here you ſee it done in the examples. Thus in this maner the roote ſ8 is extracted with ſome more breuitie, then by the two former operations. And in

```
   ℨ
  1418
 532045
83084095520768 (608
 64881636
  642

   ℨ  51
  14184 2
 5320458 3
83084095520768 (608
 64881636  3
  642

   ℨ  51
  14184 2
 5320458332
83084095520768 (608
 64881636  ℨ
  642

   ℨ  51
  14184 2
 5320458332
83084095520768 (608
 64881636  ℨ
  642
```

in like maner and many other wayes, you may extract the root ſg of any other ſg number giuen. So that if you will proceede further to extract of a number giuen any other roote required, whatſoeuer, hauing well conceiued the premiſes, you may with eaſe attaine to your purpoſe pretended, without any more examples.

The 3 Inſtruction is of the extractions of rootes in fractions.

ANy fraction giuen, being the ʒ, the ȼ, or any other product of a ȝ often taken and multiplied, hath for the numerator of his roote, the roote of the numerator of the fraction giuen, and for the denominator of his root, the roote of the denominator of the fraction giuen: euer eſteeming the numerator, and the denominator of the fraction giuen, as if each of thē apart, were the product of ſo many multiplications, as the fraction giuen is ſayd to be the product &c.

Examples.

This ʒ fraction $\frac{625}{961}$ hath $\frac{25}{31}$ his roote, and is thus extracted; the roote ʒ of the numerator is 25 and the roote ʒ of the denominator is 31, as here you ſee in the examples. This ʒȼ fraction $\frac{262144}{531441}$ hath $\frac{8}{9}$ his roote, and is thus extracted. Firſt by extracting the ʒ roote of the fraction giuen, and then the ȼ roote of that roote, euer working as the compoſition of the caracter, doth require &c. as here you ſee in the examples.

z

ʒ 625 (25 — 4

ʒ 961 (31 — 6 — *facit* $\frac{25}{31}$

x 0

ʒȼ 262144 (ȼ 512 (ȝ 8 — xooo — x

x

ʒȼ 531441 (ȼ 729 (ȝ 9 — 430 — x444 — x — *facit* $\frac{8}{9}$

The

The 4 Inſtruction is of Surd numbers, how they are expreſſed and pronounced: alſo how to find the value of them, ſo nigh as ſhall be thought neceſſarie.

THough number may be applied to expreſſe the value of any quantitie, for that any quantitie giuen, taken, or imagined, may be deuided, or imagined to be deuided; in ſo many equall parts, as any number giuen, taken, or imagined, doth containe vnits. Yet when that number firſt giuen, taken, or imagined is ſo applied to that quantitie firſt giuen, taken, or imagined, and that in the ſame operation and according to the ſame quantitie to the vnit thus firſt limited, number muſt be applied to quantities to the firſt quantitie giuen, taken, or imagined incommunicant, then in that reſpect number can not be applied to expreſſe the value of thoſe quantities to the firſt quãtitie giuen, taken, or imagined incommunicant, though the vnit ſhould be parted in the greateſt multitude of partes that can be named or imagined, and therefore thus conſidered their value is ſurd and vnpronounceable. Neuertheleſſe for the arithmeticall attaining to the quantitie vnknowen deſired, in as much as theſe ſurd quantities are certaine, true, and vnuariable, we conceiue their ſurd value in the mind, and repreſent the ſame to the ſight, by the numbers whereof the ſurd values are the roots, hauing that caracter pronounced or placed before them, that ſheweth what product of multiplications the number is taken to be, and diſtinct from other numbers by this caracter √ pronounced or placed firſt before it; which caracter √ doth euer ſignifie, that the ſuch roote of the number following, as the caracter ſignifieth, is the quantitie which we couet to expreſſe. And ſo for breuitie ſake, we call theſe ſuch repreſentations abuſiuely ſurd numbers: as √ȝ 18 ſignifieth the ſquare roote of 18, and is called roote ȝ 18. This √c 54 the cubicke roote of 54, and is called roote c 54. This √ȝc 1458 the ȝc roote of 1458, and is called roote ȝc 1458. And ſo vnderſtand of all other. And in regard that theſe ſurd numbers in their operations are diuerſely ioyned together by the ſignes +

and —, and againe diſtinct by pricke or prickes, therefore for the better methode, we call all ſurd numbers which are repreſented without the ſignes + — diſtinction of pricke or prickes, generally *ſurd mediall* numbers. Thoſe which are repreſented with the ſignes + and — without diſtinction of pricke or prickes, we call generally, *ioyned ſurd* numbers: but of theſe diſtinctiuely, thoſe which are repreſented with onely once the helpe of the ſigne + we call *binomes*, with two times we call *trinomes*, with three times we call *quadrinomes*, &c. euer agreeing to the number of the particulars by which it is expreſſed. Thoſe which are repreſented with onely once the helpe of the ſigne —, we call *reſidues* or remainers *binomes*, with two times *reſidues trinomes* &c. ſo that euery *binome*, *trinome*, *quadrinome &c.* hath his *reſidue*, by placing the ſigne — for the ſigne +. And finally all thoſe which are repreſented with diſtinctiō of pricke or prickes, we call generally, *diſtinct ſurd* numbers. Now albeit that it can neuer be exactly expreſſed, how manie vnits and parts of vnit, any of theſe *ſurd* numbers are, yet the vnit being parted in ſo many parts as ſhall be thought neceſſary, it may be knowne, how many of thoſe partes they do containe within one of them, in this order. Take the number of partes that the vnit is parted into for a roote, and multiply it as the number which helpeth to repreſent the *ſurd mediall* number is ſayd to be multiplied. Then reduce the ſayd number which helpeth to repreſent the *ſurd mediall* number into the forme of a fractiō, hauing for denominator the product produced (as aforeſayd) of the number of the parts that the vnit is parted into, and then the greateſt roote of that fraction, according to the caracter of the *ſurd mediall* number, is the value of the *ſurd mediall* nūber required, within one of thoſe partes that the vnit is parted into.

Examples.

IF the value of this *ſurd mediall* number $\sqrt{\mathfrak{Z}}$ 18 be required, part the vnit for facilitie into 100 parts, whoſe $\mathfrak{Z}$ is 10000, vnder which denomination bring 18, and make it a fraction thus $\sqrt{\mathfrak{Z}}\frac{1800000}{10000}$ whoſe greateſt roote $\mathfrak{Z}$ is $\frac{424}{100}$, that is $4\frac{24}{100}$, which

is a litle leſſe then √ʒ 18, and $4\frac{25}{100}$ is a litle more. If the value of this ſurd mediall number √ȼ 54 be required, deuide the vnit for facilitie in 10 partes, whoſe ȼ is 1000, vnder which denomination bring 54, thus √ȼ $\frac{54000}{1000}$, whoſe greateſt ȼ root is $\frac{37}{10}$, that is $3\frac{7}{10}$, which is a litle leſſe then √ȼ 54, and $3\frac{8}{10}$ is a litle more. Thus in like maner (if you thinke it conuenient, deuiding the vnit in any other number of parts) you may know very nigh the value of any ſurd number giuen, hauing alwayes good regard, what is meant to be expreſſed by the caracters, numbers, ſignes and prickes rehearſed or written. As that this ioyned ſurd number or binome √ʒ 12 + √ʒ 5 ſignifieth the aggregat of √ʒ 12 and √ʒ 5. This ioyned ſurd number or reſidue √ʒ 12 — √ʒ 5, the remainer of √ʒ 12 this √ʒ 5 being abated. This √ʒ 27 — 2 the remainer of √ʒ 27 this 2 being abated. This √ʒ 74 + √ȼ 10 — 4, the remainer of √ʒ 74 + √ȼ 10 this 4 being ſubſtracted. This √ʒ 73 — √ʒ 17 + 3, the remainer of √ʒ 73 + 3 this √ʒ 17 being ſubſtracted. This √ȼ 719 — √ʒ 8 — 1, ſignifieth the remainer of the remainer of √ȼ 719 this √ʒ 8 being ſubſtracted, & then this 1 being alſo abated out of that remainer. This diſtinct ſurd number √ʒ. √ʒ 17 + 8 hauing the two firſt caracters ſeparated for diſtinction from the other with one pricke, ſignifyeth the ʒ roote of the number following the ſame, which is of √ʒ 17 + 8. This √ȼ . √ʒ 31 — √ȼ 7 ſignifieth the ȼ roote of √ʒ 31 — √ȼ 7. This √ʒ . √ʒ 8 + 9 : + √ʒ 7 — 2 hauing alſo a diſtinction in the midſt made with two prickes, ſignifieth the aggregat of √ʒ . √ʒ 8 + 9 and √ʒ 7 — 2, and may alſo be written thus √ʒ 7 — 2 + √ʒ . √ʒ 8 + 9, the like vnderſtand of other. This √ȼ . √ʒ 179 + √ʒ 88 : — 3 ſignifieth the remainer of √ȼ . √ʒ 179 + √ʒ 88 this 3 being abated. This √ȼ . √ʒ ȼ 493 — √ʒ 13 : + √ʒ . √ȼ 87 + 12 ſignifieth the aggregat of √ȼ . √ʒ ȼ 493 — √ʒ 13 and √ʒ . √ȼ 87 + 12. This √ʒ . √ȼ . √ʒ ʒ 493 — √ʒ 13 : — √ʒ . √ʒ ʒ 1903 — 11, hauing the two firſt & the two next caracters ech of thē diſtinct from the other with one pricke, ſignifieth the ʒ roote of the remainer of √ȼ . √ʒ ȼ 493 — √ʒ 13 this √ʒ . √ʒ ʒ 1903 — 11 being ſubſtracted, &c. of all other.

The 5 Inſtruction is of abbreuiation in ſurd mediall numbers.

A Surd mediall number being giuen, to write the value of it with the leaſt number and caracter that it can be written with Conſider the compoſition of the caracter of the ſurd number giuen, and extract the ſuch roote, roote of roote, or roote of rootes, of the number which doth helpe to ſignifie the ſurd number giuen, as the caracters whereof the caracter of the ſurd number giuen is compounded requireth, and can be extracted; and then that roote placed with the reſidue of caracters of the ſurd number giuen, thoſe caracters by which you made extractions being left out, is the leaſt number and caracter that it can be written with. But if all the rootes of the ſaid number that the caracters of the compoſition of the ſaid caracter requireth be ſurd, then the ſurd number giuen cannot be abbreuiated.

Examples.

If ✓ȝ·c 27 be giuen to be abbreuiated, the ȝ· roote of 27 is ſurd, the c roote of 27 is 3, therefore ✓ȝ· 3 is equall to ✓ȝ·c 27 giuen. If ✓ȝ·c 25 be giuen to be abbreuiated, the ȝ· roote of 25 is 5, the c roote of 5 is ſurd, therefore ✓c 5 is equall to ✓ȝ·c 25 giuen. If ✓ȝ·c ſ 1024 be giuen to be abbreuiated, the ȝ· roote of 1024 is 32, the c roote of 32 is ſurd, the ſ roote of 32 is 2, therefore ✓c 2 is equall to ✓ȝ·c ſ 1024 giuen. If ✓ȝȝ·c bſ 64 be giuen to be abbreuiated, the ȝ· roote of 64 is 8, the ȝ· roote of 8 is ſurd, the c roote of 8 is 2, the b ſ roote of 2 is ſurd, therefore ✓ȝ·bſ 2 is equall to ✓ȝȝ·c bſ 64 giuen. If ✓ȝ·c 23 be giuen to be abbreuiated, the ȝ· roote of 23 is ſurd, the c roote of 23 is ſurd, therefore ✓ȝ·c 23 giuen, cannot be abbreuiated &c.

The 6 Inſtruction is to bring or reduce two ſurd mediall numbers giuen vnder one like name of caracter.

TO bring two ſurd mediall numbers giuen vnder one like name of caracter. Multiply the number which helpeth to ſignifie

ſignifie the firſt ſurd number giuen, as the caracter of the other ſurd number giuen ſignifieth the product of multiplications, & multiply the number which helpeth to ſignifie the ſecond ſurd number giuen, as the caracter of the firſt ſurd number giuen ſignifieth the product of multiplications. Then ioyne the caracters of both the ſurd numbers giuen together, placing them before the two products of the aforeſaid multiplications, with this caracter ✓ placed firſt in order before all, and then thoſe two new ſurd numbers ſo written, are equall vnto the two ſurd numbers giuē, and are reduced vnder one like name of caracter.

Examples.

If ✓ȝ 8 & ✓ȝȝ 7 be giuē to be reduced vnder one like name of caracter, firſt multiply 8 cubickly, and 7 zenſickly, the products are 5 1 2 & 4 9, before which products place the caracters of both the ſurd nūbers giuen ioyned together thus ✓ȝȝȝ 5 1 2 and ✓ȝȝȝ 4 9, which two ſurd numbers are equall vnto the two ſurd numbers giuen, and are reduced vnder one like name of caracter. But if the caracters of the ſurd numbers giuen, be compounded of many caracters ioyned together, then for more breuitie, worke as if the ſurd numbers giuē had not in the compoſition of their caracters, thoſe caracters which are common to them both: as if ✓ȝ 3 and ✓ȝȝȝ 28 be giuen to be reduced vnder one like name of caracter, multiply 3 only cubickly, and they are reduced to ✓ȝȝȝ 27 and ✓ȝȝȝ 28. If ✓ȝȝȝ 2 and ✓ȝſſ 4 be giuen to be reduced vnder one like name of caracter, multiply 2 ſurſolidly, and 4 cubickly, ſo they are reduced to ✓ȝȝȝſſ 32 and ✓ȝȝȝſſ 64. Note if ✓ȝ 7 and 2 a rationall number be giuen to be reduced vnder one like name of caracter, reduce 2 in the forme of the ſurd number giuen: and they are reduced to ✓ȝ 7 and ✓ȝ 4 &c. Sometime the two ſurd nūbers giuen may be reduced by abbreuiation, as if ✓ȝȝȝſſ 32 and ✓ȝȝȝ 6 be giuen to be reduced vnder one like name of caracter, abbreuiate onely ✓ȝȝȝſſ 32, and they are reduced to ✓ȝȝȝ 2 and ✓ȝȝȝ 6. If ✓ȝȝȝȝȝ 125 and ✓ȝȝȝȝȝ 169 be giuen to be reduced vnder one like name of caracter, abbreuiate ✓ȝȝȝȝȝ 125 to ✓ȝȝȝ 5, and abbreuiate ✓ȝȝȝȝȝ 169 to

√ʒ√ʒ 13, &c. of all other.

The 7 Inſtruction is to find the reaſon betweene two ſurd mediall numbers giuen of one kind, compared together.

TO find the reaſon betweene two ſurd mediall numbers giuen of one kind compared together. If they be not vnder one like name of caracter, reduce them to it, then find the reaſon that is betweene the numbers which helpe to expreſſe the two ſurd numbers ſo reduced or giuen, and the roote of the number that expreſſeth their reaſon, taken as the product of ſo manie multiplications as the caracters of the ſurd numbers ſo reduced or giuen ſignifie, is the reaſon required: and this reaſon is either rationall or ſurd.

Examples.

If √ʒ 2 and √ʒ 50 be giuen to haue their reaſon found, the reaſon of 2 to 50 is ſub 25 times, the √ʒ of 25 is 5, therefore the reaſon of √ʒ 2 to √ʒ 50 is ſub 5 times. If √c 108 to √c 32 be giuen to haue their reaſon found, the reaſon of 108 to 32 is $3\frac{3}{8}$ times, the √c of $3\frac{3}{8}$ or $\frac{27}{8}$ is $\frac{3}{2}$ or $1\frac{1}{2}$, therfore the reaſon of √c 108 to √c 32 is $1\frac{1}{2}$ times. If √ʒ 12 to √ʒ 24 be giuen to haue their reaſon found, the reaſon of 12 to 24 is ſub 2 times, the √ʒ of 2 is √ʒ 2, therefore the reaſon of √ʒ 12 to √ʒ 24 is ſub √ʒ 2 times. If √ʒ 7 to √c 3 be giuen to haue their reaſon found, firſt reduce them vnder one like name of caracter, and they are √ʒc 343 to √ʒc 9, the reaſon of 343 to 9 is $38\frac{1}{9}$ times, the √ʒc of $38\frac{1}{9}$ is ſurd, therfore the reaſon of √ʒ 7 to √c 3 is √ʒc $38\frac{1}{9}$ times, &c. of all other.

The 8 Inſtruction is of Addition in ſurd mediall numbers.

TO expreſſe the value of two ſurd mediall numbers giuen of one kind, in one ſurd number. Firſt if the ſurd numbers giuen be not vnder one like name of caracter reduce them to

it,

it, then conſider whether the two ſurd numbers ſo reduced or giuen, be communicants or not, if they be incommunicants, the two ſurd numbers giuen cannot be added but by this ſigne + placed betweene them: but if they be communicants, find the number that expreſſeth their reaſon, and that is a number in the ſame reaſon to 1, that the greater ſurd number giuen is to the leſſer. Therefore adde theſe two ſayd numbers together, that is, the number that nameth their reaſon and 1, and multiply their aggregat as the caracter of the ſurd numbers ſo reduced or giuen ſignifieth the product of multiplications, and multiply that product by the number that helpeth to ſignifie the leaſt ſurd number ſo reduced or giuen; then the ſuch roote of this laſt product as the caracters of the ſurd numbers ſo reduced or giuē requireth, is the aggregat of the two ſurd numbers giuen.

Examples.

If √ȝ 3 and √ȝ 2 be giuen to be added together, their reaſon is √ȝ 1 ½ times, therefore their aggregat is √ȝ 3 + √ȝ 2. If √ȝ 50 and √ȝ 2 be giuen to be added together, their reaſon is as 5 to 1, whoſe aggregat is 6, his ȝ is 36, which multiplied by 2 (the number which helpeth to ſignifie the leaſt ſurd number giuen) the product is 72, therefore √ȝ 72 is the aggregat of √ȝ 50 and √ȝ 2. If √ȸ 108 and √ȸ 32 be giuen to be added together, their reaſon is as 1 ½ to 1, whoſe aggregate is 2 ½ or ⅝ his ȸ is $\frac{125}{8}$, which multiplied by 32 the product is 500, therefore √ȸ 500 is the aggregat of √ȸ 108 and √ȸ 32. If √ȝ 8 and √ȸ 7 be giuen to be added together, reduced to one like name of caracter, they are √ȝȸ 512 and √ȝȸ 49, their reaſon is √ȝȸ 1 $\frac{23}{49}$ times, therefore their aggregate is √ȝ 8 + √ȸ 7. If √ȝȸ 512 and √ȝ 18 be giuen to be added together, reduced to one like name of caracter they are √ȝ 8 and √ȝ 18, their reaſon is as 1 to 1 ½ whoſe aggregate is 2 ½, his ȝ is 6¼, which being multiplied by 8 (the number which helpeth to ſignifie the leaſt ſurde number ſo reduced) the product is 50, therefore √ȝ 50 is the aggregat of √ȝȸ 512 and √ȝ 18, &c. of all other.

The 9 Inſtruction is of ſubſtraction in ſurd mediall numbers.

TWo ſurd mediall numbers being giuen of one kind, to find howe much remaineth, the leſſer being abated out of the greater. Worke in all things as if they ſhould be added together, except in place where you placed this ſigne + there you muſt place this ſigne —, alſo in that place of addition where you added the two aforeſaid numbers which expreſſe their reaſon, there you muſt heere ſubſtract the leſſer out of the greater.

Examples.

If √ȝ 2 to be abated out of √ȝ 3 be giuen, their reaſon is ſurd, therfore √ȝ 3 — √ȝ 2 is the remainer. If √ȼ 3 to be abated out of √ȝ 7 be giuen, they are incommunicants, therfore √ȝ 7 — √ȼ 3 is the remainer. If √ȝ 2 to be abated out of √ȝ 50 be giuen, their reaſon is as 1 to 5, ſubſtract 1 out of 5 remaineth 4, his ȝ is 16, which being multiplied by 2, (the nūber which helpeth to ſignifie the leaſt ſurd number giuen) the product is 32, therefore √ȝ 32 is the remainer. If √ȼ 3 to be abated out of √ȼ 24 be giuen, their reaſon is as 1 to 2, ſubſtract 1 out of 2 remaineth 1, his ȼ is 1, which being multiplied by 3, the product is 3, therefore √ȼ 3 is the remainer, and ſo of all other.

The 10 Inſtruction is of Multiplication in ſurd mediall numbers.

TWo ſurd mediall numbers being giuē of what kind ſoeuer, to finde a third ſurd number, or if it ſo happen a rationall number, that ſhall containe the one of them of the ſame kinde, ſo many times as the other doth containe vnits. If the two ſurd numbers giuen be not vnder one like name of caracter, reduce them to it, then multiply the numbers together that helpe to repreſent the ſurd numbers ſo reduced or giuen, and the ſuch root of their product as the caracters of the ſurd numbers ſo reduced

duced or giuen requireth, is the product desired.

Examples.

If √ʒ 12 to be multiplied by √ʒ 3 be giuen, multiply 12 by 3, the product is 36, whose ʒ roote is 6, therfore 6 is the product demaunded. If √ʒ 5 be giuen to be multiplied by √ʒ 5, the product is 5, and requireth no other operation. If √ʒ 24 to be multiplied by √ʒ 7 be giuen, multiply 24 by 7, the product is 168, therfore √ʒ 168 is the product desired. If √cℓ 30 to be multiplied by √cℓ 9 be giuen, multiply 30 by 9, the product is 270: therefore √cℓ 270 is their product. If √cℓ 16 to be multiplied by √cℓ 4 be giuen, multiply 16 by 4, the product is 64, whose cℓ roote is 4, therefore their product is 4. If √ʒ 7 to be multiplied by √cℓ 3 be giuen, reduce them to one like name of caracter, & they are √ʒcℓ 343 and √ʒcℓ 9, and their product is √ʒcℓ 3087 &c. of all other.

The 11 Instruction is of Deuision in surd mediall numbers.

TWo surd mediall numbers being giuē of one kind, to finde how many times the lesser is contained in the greater, is no more but to finde the number that nameth their reason. But if the lesser surd number giuen be to be deuided by the greater, the operation in effect is the same, but that the quotient is a fraction, either rationall or surd.

Examples.

If √ʒ 3 to be deuided by √ʒ 2 be giuen, their quotient is √ʒ $1\frac{1}{2}$. If √ʒ 50 to be deuided by √ʒ 2 be giuen, their quotient is √ʒ 25, that is 5. If √ʒ 7 to be deuided by √cℓ 3 be giuen, reduce them to one like name of caracter, and they are √ʒcℓ 343 and √ʒcℓ 9, their quotient is √ʒcℓ $38\frac{1}{9}$. If √ʒ 2 to be deuided by √ʒ 3 be giuen, the quotient is √ʒ $\frac{1}{1}$. If √ʒ 2 to be deuided by √ʒ 50 be giuen, the quotient is √ʒ $\frac{1}{25}$, that is $\frac{1}{5}$. If √cℓ 3 to be deuided by √ʒ 7 be giuen, reduce them to one like name of caracter, & the quotient is √ʒcℓ $\frac{9}{343}$ &c. of all other.

The 12 Inſtruction is of additions of the ſignes + and —

FIrſt vnderſtand that all quantities which are written without the ſignes + or —, are alwaies eſteemed as if they had the ſigne + placed before them. Then know, that + added vnto + is more, and — added vnto — is —, but to adde + vnto — or — vnto + you muſt ſubtract the quantities the leſſer out of the greater, and the remainer muſt haue the ſigne of the greater quantitie: and then that remainer ſo ſigned is their aggregate. In this there is no difficulty, yet for your more perceiuerance we will make the example in rationall numbers.

Examples.

to adde 13 + 8	to adde 13 — 8	to adde 13 + 8
and 9 + 5	and 9 — 5	and 9 — 5
facit 22 + 13	*facit* 22 — 13	*facit* 22 + 3
to adde 13 — 8	to adde 13	to adde 23 — 12 + 7
and 9 + 5	and — 9	and 14 + 10 — 9
facit 22 — 3	*facit* 4	*facit* 37 — 2 — 2

The 13 Inſtruction is of ſubſtractions of the ſignes + and —

IF the quantitie to be abated be leſſe then the quantitie of which it muſt be abated, then + abated out of + remaineth +, and — abated out of — remaineth —. But if the quantitie to be abated be greater then the quantitie out of which it muſt be abated, yet the leſſer quantitie muſt be ſubſtracted out of the greater; but the remainer muſt haue the contrarie ſigne of the quantities giuen. Alſo if + muſt be abated out of —, or — out of + then the quantities giuen, muſt be added together, & their aggregate muſt haue the ſigne of the quantitie, out of which you do make the ſubſtraction, and that aggregate ſo ſigned is the remainer deſired. This needeth no other explication then the

the examples following.

Examples.

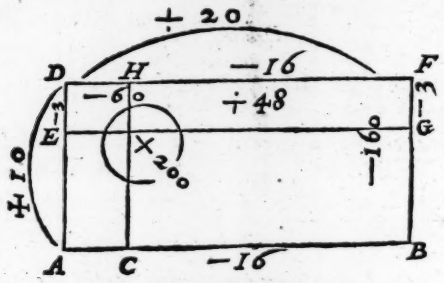
from 13 + 8	from 13 — 8	from 13 + 5
abate 9 + 5	abate 9 — 5	abate 9 + 8
facit 4 + 3	*facit* 4 — 3	*facit* 4 — 3
from 13 — 5	from 13 + 8	from 23 — 8
abate 9 — 8	abate 9 — 5	abate 9 + 5
facit 4 + 3	*facit* 4 + 13	*faeit* 14 — 13

The 14 Inſtruction is of multiplications of the ſignes + and —

+ multiplyed by +, or — multiplied by —, the product is +, but + multiplied by —, or — multiplied by +, the product is —. Though ſome part of theſe operations of the ſignes + and —, at the firſt ſhew, ſeeme to haue no ground of reaſon: yet after that the examples are conceiued (which for your better vnderſtanding we will make aſwell by geometricall demenſions, as in rationall numbers) the effects will be very apparent.

Examples.

If the line A B — B C, to be led in or multiplied by the line A D — E D, be the lines giuen: it is apparent that the parallelogram rectangle A F, is the product of A B led in or multiplied by A D, the parallelogram rectangle B H, the product of B C led in B F or A D for that they are equall. The parallelogram rectangle E F, the product of E D led in D F or A B for that they are equall. And the parallelogram rectangle G H, the product of G F or D E for that they are equall led in F H or B C for that they are equall. And the parallelogram rectangle C E, is the product deſired of the vnknowne lines A C led in A E giuen. Now the lines A B and A D knowne, the parallelogram A F is knowne. The lines B C and A D knowne, the parallelogram C F is knowne. The lines D E and A B knowne, the parallelogram E F is knowne. The lines D E



and B C knowne, the parallelogram G H is knowne. And ſo conſequently by additions and ſubſtractions, the parallelogram C E deſired is knowne. For if the parallelogram E F be ſubſtracted out of the parallelogram A F, the remainer is the parallelogram A G, and if the parallelogram A G haue the parallelogram G H added vnto it, and from that aggregate the parallelogram C F being abated, the remainer is the deſired parallelogram C E, which then is alſo knowne. Thus it is manifeſt that the parallelogram + A F is produced by the multiplicatiõ of + A B by + A D, the parallelogram — E F is produced by the multiplication of + A B by — D E, the parallelogram + G H is produced of — B C by — D E, and the parallelogram — B H is produced by the multiplication of — B C by + A D, the explaining of which operations of the ſignes + and — was our intent. In rationall numbers, the example ſtandeth thus. If 20 — 16 to be multiplied by 10 — 3 be giuen. Firſt multiplie + 20 by + 10 the product is 200, then multiplie — 16 by the ſame + 10 the product is — 160: then multiply + 20 by — 3, the product is — 60, and laſtly multiply — 16 by the ſame — 3 the product is + 48, nowe all theſe products added and ioyned by the ſignes + and —, are 200 — 220 + 48 which in effect is but 28, which is alſo the product of 20 — 16 which is 4, multiplied by 10 — 3 which is 7.

20 — 16
10 — 3

200 — 160
— 60 + 48

facit 200 — 220 + 48

Theſe two examples well conceiued and vnderſtood do not onely giue perfect knowledge of the multiplications of the ſignes + and —, but alſo of their deuiſions, and extractions of their rootes, &c.

The 15 Inſtruction is of deuiſions of the ſignes + and —

+ deuided by + or — deuided by — the quotient is +, but + deuided by —, or — deuided by + the quotient is —. This is eaſie to be conceaued by that which is aforeſaid of multiplications, and therefore it ſhall neede no farther explication then the examples and their worke.

Examples.

If 200 — 220 + 48 be giuen to be deuided by 10 — 3 the quotient is 20 — 16. And if — 30 + 80 — 40 be giuen to be deuided by — 5 + 10 the quotient is 6 — 4. You muſt place the ſaid deuiſions, and worke them as heere you ſee done in the examples following, and needeth no other declaration.

16	2
20 — 220 + 48 (20 — 16	— 30 + 80 — 40 (6 — 4
200 — 3 — 3	— 5 + 10 + 10
+ 160	— 5

The 16 Inſtruction is of addition in ioyned ſurd numbers.

TO expreſſe the quantities of many ioyned ſurde numbers giuen of one kinde, in one quantitie. Make many

additions, or additions & ſubſtractions of the perticulers of the ioyned quantities giuen, according as the ſignes + or — giue cauſe to be done, and the ſuch aggregates, or aggregats and remainers, ioyned by the ſuch ſignes + or — as they require to haue, is the aggregate deſired. This needeth no other explication then the examples following.

Examples.

$9 + \sqrt{24}$
$7 + \sqrt{6}$
facit $16 + \sqrt{54}$

$\sqrt{180} + \sqrt{48}$
$\sqrt{125} + \sqrt{27}$
facit $\sqrt{605} + \sqrt{147}$

$\sqrt{75} + 2$
$\sqrt{12} - 3$
facit $\sqrt{147} - 1$

$\sqrt{405} + \sqrt{64}$
$\sqrt{216} - \sqrt{405}$
facit $\sqrt{1000}$

$\sqrt{216} - \sqrt{405}$
$\sqrt{64} - \sqrt{80}$
fac: $\sqrt{1000} - \sqrt{3125}$

$\sqrt{256} - \sqrt{27}$
$\sqrt{81} + \sqrt{8}$
fa: $\sqrt{2401} - \sqrt{1}$

$\sqrt{80} + \sqrt{63} - \sqrt{17}$
$\sqrt{13} - \sqrt{11} + \sqrt{15}$
fa: $\sqrt{80} + \sqrt{13} + \sqrt{63} + \sqrt{15} - \sqrt{11} - \sqrt{17}$

The 17 Inſtruction is of ſubſtraction in ioyned ſurd numbers.

TWo ioyned ſurd numbers being giuen of one kinde, to finde how much remaineth the leſſer being ſubſtracted out of the greater. Abate or adde the perticulers of the ſurd nũbers giuen, as the ſignes + or — giue cauſe to be done, and the ſuch remainers, or remainers and aggregates ioyned by the ſignes + or — as they require to haue, is the remainer deſired. This needeth no other declaration, then the examples following.

Examples.

Examples.

16 + √3 54
9 + √3 6
facit 7 + √3 24

√3 605 + √3 147
√3 180 + √3 48
facit √3 125 + √3 27

√3 147 − 1
√3 75 + 2
facit √3 12 − 3

√33 2401 − √c 1
√33 256 − √c 27
fac: √33 81 + √c 8

√c 1000 − √33 3125
√c 216 − √33 405
facit √c 64 − √33 80

√33 2401 + √c 1
√33 81 + √c 8
fac: √33 256 − √c 1

9 + √3 50 − √c 21
√3 32 − 5 + √33 7
facit 14 + √3 2 − √c 21 − √33 7

√3 80 + √c 63 − √33 17
√3 13 − √c 11 + √33 15
fa: 3 80 + √c 63 + √c 11 − √3 13 − √33 17 − √33 15

The 18 Inſtruction is of multiplication in ioyned ſurd numbers.

TWo ioyned ſurd numbers being giuen, of what kinde ſoeuer, to finde a third ſurd number, or if it ſo happen a rationall number of the one kinde; that ſhall containe the one of them of the ſame kinde, ſo many times as the other doth containe vnits. Multiplie all the particulers of the one ſurde number giuen, ſeuerally and in order one after other, by all the ſeuerall particulers of the other ſurde number giuen, in order one after other, and the aggregate of all thoſe ſeuerall products is the ſurde number or rationall number deſired: as by the examples following is made manifeſt.

Examples.

$8+\sqrt{3}45$
$6+\sqrt{3}20$

$48+\sqrt{3}1620$
$+\sqrt{3}1280+\sqrt{3}900$

facit $78+\sqrt{3}5780$

Note that a rationall nũber muſt be reduced in the forme of the ſurd nũber that he is multiplied by.

$8+\sqrt{3}45-\sqrt{cc}8$
$6-\sqrt{3}20$

$48+\sqrt{3}1620-\sqrt{cc}1728$
$-\sqrt{3}1280-\sqrt{3}900+\sqrt{3cc}512000$

facit $6+\sqrt{3}180$

$\sqrt{33}288-\sqrt{33}648$
$\sqrt{33}128-\sqrt{33}162$

$\sqrt{333}36864-\sqrt{333}82944$
$-\sqrt{333}46656+\sqrt{333}104976$

facit $18+\sqrt{3}192-\sqrt{3}288-\sqrt{3}216$

If $6+\sqrt{3}12$ and his reſidue $6-\sqrt{3}12$ be the two ſurd numbers giuen, to be multiplied together. 6 multiplied by $+\sqrt{3}12$, and $-\sqrt{3}12$ multiplyed by 6, the one product doth ballance the other, therefore of thoſe ſuch multiplications the one and the other is vnneceſſarie to be made: the like vigilant regarde you muſt haue in all operations where the ſigne + doth ballance the ſigne — &c.

$6+\sqrt{3}12$
$6-\sqrt{3}12$

$36 + 12$

facit 48

If $6+\sqrt{3}14$ and $6+\sqrt{3}14$ be the two ſurde numbers giuen to be multiplied together. Foraſmuch as that we muſt multiplie 6 by $\sqrt{3}14$ and againe $\sqrt{3}14$ by 6, which two multiplications haue equall products, and that thoſe two ſaid products muſt alſo be added together, therefore it is a better way to double one of the perticulers, to ſay either 6 or $\sqrt{3}14$, and by that double to multiply that other perticuler, whoſe product is equall vnto the aggregate of the two other products aforeſaid, &c. And the like conſiderations you muſt haue

$6+\sqrt{3}14$
$6+\sqrt{3}14$

fa: $50+\sqrt{3}2016$

$\sqrt{3}14-\sqrt{3}7$
$\sqrt{3}14-\sqrt{3}7$

fac: $21-\sqrt{3}392$

if $\sqrt{3}\,14 - \sqrt{3}\,7$ be giuen to be multiplied by it ſelf, and alſo in all other operations, to attaine the next way vnto the thing deſired. If $3+\sqrt{3}\,12+\sqrt{3}\,8$ & this reſidue $3+\sqrt{3}\,12-\sqrt{3}\,8$ be the two ſurd numbers giuen to be multiplied together. Firſt multiplie $3+\sqrt{3}\,12$ by $3+\sqrt{3}\,12$, the product is $21+\sqrt{3}\,432$, then multiply $-\sqrt{3}\,8$ by $+\sqrt{3}\,8$ the product is -8 and of all the reſt euer the + doth ballance the $-$ &c. If $4-\sqrt{3}\,2$ be giuen to be multiplied cubick, the operation ſtandeth as heere you ſee it placed. But if you vnderſtand what we haue written in the 2 inſtruction of this booke, and with a ſpeculatiue minde, taking each perticuler of your ſurde number giuen, for one figure of your roote, as you muſt do: then the operation is as heere you ſee it placed, and is wrought as followeth. The ☌ of 4 is 64, the ☌ of $-\sqrt{3}\,2$ is $-\sqrt{3}\,8$, the product of the triple of 4 multiplied by the ☽ of $-\sqrt{3}\,2$, is $+24$, the product of the triple of the ☽ of 4 multiplied, by $-\sqrt{3}\,2$ is $-\sqrt{3}\,4608$, all which particulers are $88-\sqrt{3}\,5000$, the ☌ deſired. The operation to finde the ☌ of $4+\sqrt{3}\,2$, is as heere you ſee placed. And if you ſpeculate further of the ☌ of the ſaid binome and his reſidue afore giuen, well marking that the ☽ of the firſt particuler, and the product of the ☽ of the ſecond particuler multiplied by 3, are both multiplied by the firſt particuler.

$$\begin{array}{r} 3+\sqrt{3}\,12+\sqrt{3}\,8 \\ 3+\sqrt{3}\,12-\sqrt{3}\,8 \\ \hline 21+\sqrt{3}\,432-8 \\ \textit{facit}\ 13+\sqrt{3}\,432 \end{array}$$

$$\begin{array}{r} 4-\sqrt{3}\,2 \\ 4-\sqrt{3}\,2 \\ \hline 18-\sqrt{3}\,128 \\ 4-\sqrt{3}\,2 \\ \hline 72-\sqrt{3}\,2048 \\ -\sqrt{3}\,648+\sqrt{3}\,256 \\ \hline \textit{fa:}\ 88-\sqrt{3}\,5000 \end{array}$$

$$\begin{array}{r} 4-\sqrt{3}\,2 \\ 4-\sqrt{3}\,2 \\ \hline 64-\sqrt{3}\,8 \\ 24-\sqrt{3}\,4608 \\ \hline \textit{facit}\ 88-\sqrt{3}\,5000 \end{array}$$

$$\begin{array}{r} 4+\sqrt{3}\,2 \\ 4+\sqrt{3}\,2 \\ \hline 64+\sqrt{3}\,8 \\ 24+\sqrt{3}\,4608 \\ \hline \textit{facit}\ 88+\sqrt{3}\,5000 \end{array}$$

Then againe, that the ☽ of the ſecond particuler, and the product of the ☽ of the firſt perticular multiplyed by 3, are both multiplied by the ſecond particuler.

$$\begin{array}{r} 4-\sqrt{3}\,2 \\ \hline 16 \quad 2 \\ 6 \quad 48 \\ \hline 22 \quad 50 \\ \textit{fa:}\ 88-\sqrt{3}\,5000 \end{array} \qquad \begin{array}{r} 4+\sqrt{3}\,2 \\ \hline 16 \quad 2 \\ 6 \quad 48 \\ \hline 22 \quad 50 \\ \textit{fa:}\ 88+\sqrt{3}\,5000 \end{array}$$

Your operation will manifeſtly appeare to be yet in better manner, as you ſee them heere placed.

The 19 Inſtruction is of deuiſion in ioyned ſurde numbers.

TWo ioyned ſurd numbers of one kinde, being giuen to finde how many times the leſſer is contained in the greater. Place the firſt particuler of the deuider, vnder the firſt particuler of the deuident, and ſo the other in order, the next vnder the next &c. Then deuide the firſt particuler of the deuident, by the firſt particuler of the deuider, ſtriking the figures out, and leauing no remainer ouer the ſame: with this quotient multiplie all the other ſeuerall perticulers of the deuider, abating each ſeuerall product out of the ſeuerall particuler ſtanding ouer it: ſtriking out and writing their ſeuerall remainers ouer them. Then place your deuider againe a new, and worke as before is ſaid, and ſo continue to the end, and then the quotient will expreſſe your deſire.

Examples.

If $48 - \sqrt{3}\ 1280 - \sqrt{3}\ 1620 + 30$ be giuen to be deuided by $6 - \sqrt{3}\ 20$. The worke of this deuiſion is as heere you ſee in the example.

| | | |

ø	1	20	22
1122	1429	612	779
$\sqrt{3}\sqrt{3}36864$	$\sqrt{3}\sqrt{3}46656$	$\sqrt{3}\sqrt{3}82944$	$\sqrt{3}\sqrt{3}104976$
$\sqrt{3}$ 192	$-\sqrt{3}$ 216	$-\sqrt{3}$ 288	$+$ 18 .
$\sqrt{3}\sqrt{3}128$	$-\sqrt{3}\sqrt{3}$ 162	$\sqrt{3}\sqrt{3}128$	$-\sqrt{3}\sqrt{3}162$

facit $\sqrt{3}\sqrt{3}288 - \sqrt{3}\sqrt{3}648$

Thus you may worke all ſuch like deuiſions. But for aſmuch as theſe ſurde numbers afore giuen, ſeeme rather to be made of purpoſe, then to fall by caſualtie, and beſide that in all deuiſions giuen, this operation will not worke the effect deſired. Therefore aſwell in theſe as in all other giuen, if your deuider be a binome, multiply it by his reſidue, if it be a reſidue, multiply it by his binome, and by this product deuide the product of the deuident, multiplied by the ſame binome or reſidue that the deuider is multiplyed by, and the quotient will expreſſe your deſire. As if $66 - \sqrt{3}2000$ be giuen to be deuided by $8 - \sqrt{3}45$. The operation is as heere followeth.

$8 - \sqrt{3}45$	$66 - \sqrt{3}2000$
$8 + \sqrt{3}45$	$8 + \sqrt{3}45$
64	$528 - \sqrt{3}128000$
-45	$+ \sqrt{3}196020 - \sqrt{3}90000$
19 the new deuider.	$228 + \sqrt{3}7220$ the new deuident.

$$\begin{array}{l} \overset{3}{} \\ 228 + \sqrt{3}7220 \,(12 + \sqrt{3}20 \\ 19 \quad \sqrt{3}361 \end{array}$$

Alſo if 12 be giuen to be deuided by $\sqrt{3}10 + \sqrt{3}8$. The worke is as heere you ſee.

$\sqrt{3}10 + \sqrt{3}8$	12
$\sqrt{3}10 - \sqrt{3}8$	$\sqrt{3}10 - \sqrt{3}8$
10	$\sqrt{3}1440 - \sqrt{3}1152$ the new deuident.
-8	
2 the new deuider.	

$$\sqrt{3}\,{}^{2}\,440 - \sqrt{3}\,{}^{33}\,1152 \,(\sqrt{3}\,360 - \sqrt{3}\,288$$
$$\sqrt{3}\ 4 \qquad +\sqrt{3}\ 4$$

Heere is to be noted, that when the deuident giuen, can conueniently be deuided by the new found deuider: then it is the better way to deuide the ſame by it, and to multiply that quotient by the ſame quantitie or quantities that you haue multiplied your deuider by. As in this laſt example deuide 12 (the deuident giuen) by 2 (the new found deuider) the quotient is 6, then multiply that 6 by $\sqrt{3}\,10 - \sqrt{3}\,8$ (the quantitie that the deuider giuen is multiplied by) the product is $\sqrt{3}\,360 - \sqrt{3}\,288$, which doth ſhew that $\sqrt{3}\,10 + \sqrt{3}\,8$ (the deuider giuen) is contained in 12 (the deuident giuen) this $\sqrt{3}\,360 - \sqrt{3}\,288$ times. As heere you ſee in the example.

$\sqrt{3}\,10 + \sqrt{3}\,8$ 12 (6
$\sqrt{3}\,10 - \sqrt{3}\,8$ 2

10
−8

2 the new deuider.

$\sqrt{3}\,10 - \sqrt{3}\,8$
$\sqrt{3}\,36$

facit $\sqrt{3}\,360 - \sqrt{3}\,288$
which is the quotient deſired.

In like manner, if your deuider giuen be a trinome or quadrinome &c: you muſt vſe the means by many multiplications to bring it to be of one particuler, alwaies reſpecting that you multiplie your aforeſaid quotient found, or your deuident giuen, by the ſame quantities, that you haue multiplied your deuider by &c: but foraſmuch as theſe deuiſions be not commonly in vſe, we will onely ſhew the operations to bring the deuiders to one particuler, and the reſt of the worke being eaſie to conceiue, we leaue to thy diſcretion.

If $5 + \sqrt{3}\,8 + \sqrt{3}\,6$ be the deuider giuen, you muſt worke as heere you ſee it done.

$5 + \sqrt{3}\,8 + \sqrt{3}\,6$
$5 + \sqrt{3}\,8 - \sqrt{3}\,6$

$27 + \sqrt{3}\,800$
$27 - \sqrt{3}\,800$

729 − 800

the new deuider − 71

If

If $5+\sqrt{3}8+\sqrt{3}6+\sqrt{3}3$ be the deuider giuen, you muſt worke as heere you ſee done. And ſo of all other.

$$5+\sqrt{3}8+\sqrt{3}6+\sqrt{3}3$$
$$5+\sqrt{3}8-\sqrt{3}6-\sqrt{3}3$$
$$24+\sqrt{3}800-\sqrt{3}72$$
$$24+\sqrt{3}392$$
$$24-\sqrt{3}392$$
$$576 - 392$$

184 the new deuider.

The 20 Inſtruction is of addition in diſtinct ſurde numbers.

TO expreſſe the value of two diſtinct ſurde numbers giuen of one kinde, in one ſurde number. If the diſtinct ſurde numbers giuen be communicants, adde them in the ſame manner that you adde ioyned ſurde numbers. But if they be incommunicants, adde them by the ſigne +, and then the $\sqrt{3}$ roote of the $\sqrt{3}$ of that ſuch aggregate, is the aggregat of the two diſtinct ſurde numbers giuen.

Examples.

If $\sqrt{3}.\sqrt{3}6+\sqrt{3}2$ and $\sqrt{3}.\sqrt{3}96+\sqrt{3}32$ be the two diſtinct ſurde numbers giuen to be added together. Their reaſon is as 1 to 2, therefore their aggregate is $\sqrt{3}.\sqrt{3}486+\sqrt{3}162$. As heere you ſee in the example.

$$\sqrt{3}.\sqrt{3}96+\sqrt{3}32\ (\sqrt{3}.\sqrt{3}16\ (2$$
$$\sqrt{3}.\sqrt{3}6+\sqrt{3}2 \qquad 1 \text{ to } 2$$

3

$$\sqrt{3}.\sqrt{3}6+\sqrt{3}2$$
$$\sqrt{3}.\sqrt{3}81$$
$$\textit{facit}\ \sqrt{3}.\sqrt{3}486+\sqrt{3}162$$

If $\sqrt{3}.12+\sqrt{3}6$ and $\sqrt{3}.12-\sqrt{3}6$ incommunicants, be the two diſtinct ſurd numbers giuen to be added together: adde them firſt hy the ſigne + and then multiply their aggregate by it ſelfe, and the roote $\sqrt{3}$ of the product is your deſire. As heere you ſee it done in the example &c: of all other.

$$\begin{array}{l}\sqrt{3}.12+\sqrt{3}6:+\sqrt{3}.12-\sqrt{3}6\\ \sqrt{3}.12+\sqrt{3}6:+\sqrt{3}.12-\sqrt{3}6\\ \hline 12+\sqrt{3}6+12-\sqrt{3}6+\sqrt{3}138+\sqrt{3}138\end{array}$$

facit $\sqrt{3}.24+\sqrt{3}552$

The 21 Inſtruction is of ſubſtraction in diſtinct ſurd numbers.

TWo diſtinct ſurde numbers of one kinde being giuen, to finde how much remaineth, the leſſer being abated out of the greater. If the diſtinct ſurde numbers giuen be communicants, ſubſtract them in the ſame manner that you ſubſtract ioyned ſurd numbers. But if they be incommunicants ſubſtract them by the ſigne —, and then the 3 roote of the 3 of that ſuch remainer, is the remainer of the two diſtinct ſurd numbers giuen, the leſſer being abated out of the greater.

Examples.

If $\sqrt{3}.\sqrt{3}6-\sqrt{3}2$ be giuē to be abated out of $\sqrt{3}.\sqrt{3}486-\sqrt{3}162$ Their reaſon is as 1 to 3, therfore the remainer is $\sqrt{3}.\sqrt{3}96-\sqrt{3}32$. As heere you ſee in the example.

$$\begin{array}{l}\sqrt{3}.\sqrt{3}486-\sqrt{3}162(\sqrt{3}.\sqrt{3}81(3\\ \sqrt{3}.\sqrt{3}\ 6-\sqrt{3}\ 2\qquad 1\ \text{to}\ 3\\ \qquad\qquad\quad 2\\ \hline \sqrt{3}.\sqrt{3}6-\sqrt{3}2\\ \sqrt{3}.\sqrt{3}16\\ \hline \textit{facit}\ \sqrt{3}.\sqrt{3}96-\sqrt{3}32\end{array}$$

If $\sqrt{3}.12-\sqrt{3}6$ to be abated out of $\sqrt{3}.12+\sqrt{3}6$ incommunicants, be giuen, ſubſtract them firſt by the ſigne — then multiply that remainer by it ſelfe, and the roote 3 of the product is your deſire. As heere you ſee it done in the example, and ſo of all other.

$\sqrt{3}.12+\sqrt{3}6 : -\sqrt{3}.12-\sqrt{3}6$
$\sqrt{3}.12+\sqrt{3}6 : -\sqrt{3}.12-\sqrt{3}6$

$12+\sqrt{3}6+12-\sqrt{3}6-\sqrt{3}138-\sqrt{3}138$
facit $\sqrt{3}.24-\sqrt{3}552$

The 22 Inſtruction is of multiplication in diſtinct ſurd numbers.

THe multiplication in diſtinct ſurd numbers, hath nothing that differeth from the multiplications in ioyned ſurde numbers, and other multiplications made heeretofore: for in the one and in the other, the particulers which are to be multiplied together, muſt firſt be reduced vnder one like name of caracter. And then good and vigilant regarde being had, aſwell to the ſignes + and — as alſo to all other things in this caſe needfull, you cannot faile. Yet for your more exerciſe and greater perfection, we will ſet downe the multiplications following.

Examples.

If $\sqrt{3}.13-\sqrt{3}7$ be giuen to be multiplied by 4. Firſt when you multiply $\sqrt{3}.13$ by 4, the ſaid 4 muſt be reduced to $\sqrt{3}.16$, and when you multiplie $\sqrt{3}.\sqrt{3}7$ by 4, the ſaid 4 muſt be reduced to $\sqrt{3}.\sqrt{3}256$. As heere you ſee in the example. If $\sqrt{3}.17+\sqrt{3}5$ be giuen to bee multiplyed by $\sqrt{3}7-\sqrt{3}2$. Firſt reduce $\sqrt{3}7-\sqrt{3}2$ in the forme of a diſtinct ſurde number, which is done by the multiplication of the ſame by it ſelfe &c. As heere you ſee in the example.

$\sqrt{3}.13 \quad \sqrt{3}7$
$\sqrt{3}.16 \quad \sqrt{3}.\sqrt{3}256$
facit $\sqrt{3}.208-\sqrt{3}1792$

289

$\sqrt{3}7-\sqrt{3}2$ $\quad$ $\sqrt{3}.17+\sqrt{3}5$
$\sqrt{3}7-\sqrt{3}2$ $\quad$ $\sqrt{3}.9-\sqrt{3}56$
$\sqrt{3}.9-\sqrt{3}56$ $\quad$ *facit* $\sqrt{3}.153+\sqrt{3}405-\sqrt{3}16184-\sqrt{3}280$

If $\sqrt{c}.\sqrt{3}26+5 : -\sqrt{c}.\sqrt{3}26-5$ be giuen to be multiplied cubicke. Worke as heere after you ſee placed, which

in effect differeth little or nothing from the ſuch multiplicati-ons of the binomes &c:

$$\frac{\sqrt{c}\,.\,\sqrt[3]{26+5} : -\sqrt{c}\,.\,\sqrt[3]{26-5}}{\sqrt[3]{26+5} : \qquad\qquad \sqrt[3]{26-5}}$$

The cubes of the two parts, whoſe value is 10.

$$\frac{\begin{array}{r}\sqrt{c}\,.\,\sqrt[3]{26+5} : -\sqrt{c}\,.\,\sqrt[3]{26-5} \\ \sqrt{c}\,.\,\sqrt[3]{26+5} : -\sqrt{c}\,.\,\sqrt[3]{26-5}\end{array}}{\begin{array}{cc}\sqrt{c}\,.\,51+\sqrt[3]{2600} : & +\sqrt{c}\,.\,51-\sqrt[3]{2600} \\ 3 & 3\end{array}}$$

$$\sqrt{c}\,.\,1377+\sqrt[3]{1895400} : +\sqrt{c}\,.\,1377-\sqrt[3]{1895400}$$

The ſeuerall $\sqrt[3]{}$ of the two parts, and their ſeuerall products being multiplyed by 3.

$$\begin{array}{r}\sqrt{c}\,.\,1377-\sqrt[3]{1895400} \\ \sqrt{c}\,.\,\sqrt[3]{26+5}\end{array}$$

$$\sqrt{c}\,.\,\sqrt[3]{49299354}-\sqrt[3]{49280400}+6885-\sqrt[3]{47385000}$$

The product of the $\sqrt[3]{}$ of the laſt part multiplied by 3 multiplyed by the firſt part, which ſaid product by reaſon of his communicant particulers, is reduced to $\sqrt{c}\,.\,\sqrt[3]{18954}-135$.

$$\begin{array}{r}\sqrt{c}\,.\,1377+\sqrt[3]{1895400} \\ -\sqrt{c}\,.\,\sqrt[3]{26-5}\end{array}$$

$$-\sqrt{c}\,.\,\sqrt[3]{49299354}+\sqrt[3]{49280400}-6885-\sqrt[3]{47385000}$$

The product of the $\sqrt[3]{}$ of the firſt part multiplied by 3 multiplied by the laſt part, which ſaid product by reaſon of his communicant particulers, is reduced to $-\sqrt{c}\,.\,\sqrt[3]{18954}+135$. So that the c deſired is $10+\sqrt{c}\,.\,\sqrt[3]{18954}-135 : -\sqrt{c}\,.\,\sqrt[3]{18954}+135$. And thus of all other.

The

The 23 Inſtruction is of deuiſion in diſtinct ſurd numbers.

THe deuiſion in diſtinct ſurd numbers, hath nothing that in effect differeth from the deuiſions in ioyned ſurd numbers, and other deuiſions made heeretofore. Yet for your greater furtherance we will ſet downe the deuiſions following.

Examples.

If $\sqrt{3}.\sqrt{3}\,456-\sqrt{3}\,152$ be giuen, to be deuided by $\sqrt{3}.\sqrt{3}\,6-\sqrt{3}\,2$. Worke as heere you ſee in the example.

$$\begin{array}{l} \overset{3}{\sqrt{3}.\sqrt{3}\,456-\sqrt{3}\,152}\,(\sqrt{3}.\sqrt{3}\,76 \\ \sqrt{3}.\sqrt{3}\;\;6\;\;-\sqrt{3}\;\;2 \end{array}$$

If $\sqrt{3}\,432+\sqrt{3}\,7776$ be giuen to be deuided by 6. Firſt reduce 6 to $\sqrt{3}.36$ &c: as heere you ſee in the example.

$$\begin{array}{l} \overset{7}{\sqrt{3}.432+\sqrt{3}\,7776}\,(\sqrt{3}.12+\sqrt{3}\,6 \\ \sqrt{3}.36\;\sqrt{3}.\sqrt{3}\,1296 \end{array}$$

If $\sqrt{3}.588+\sqrt{3}\,34848$ be giuen to be deuided by $\sqrt{3}.12+\sqrt{3}\,8$. Firſt multiply your deuider and your deuident each particulerly by $\sqrt{3}.12-\sqrt{3}\,8$ &c: as heere you ſee in the example.

$$\begin{array}{l} \sqrt{3}.588+\sqrt{3}\,34848 \\ \sqrt{3}.\;12-\sqrt{3}\,8 \\ \hline \sqrt{3}.7056+\sqrt{3}\,501811\,2 \\ \qquad\qquad -\sqrt{3}\,2765952-\sqrt{3}\,278784 \\ \hline \sqrt{3}.6528+\sqrt{3}\,3329\,28 \text{ deuident.} \end{array}$$

$$\begin{array}{l} \sqrt{3}.12+\sqrt{3}\,8 \\ \sqrt{3}.12-\sqrt{3}\,8 \\ \hline \sqrt{3}.136 \text{ deuider.} \end{array}$$

$$\begin{array}{l} \overset{208\qquad 147\;\;6}{\sqrt{3}.6528+\sqrt{3}\,3329\,28}\,(\sqrt{3}.48+\sqrt{3}\,18 \\ \sqrt{3}.136\;\sqrt{3}.\sqrt{3}\,184\,96 \end{array}$$

The 24 Inſtruction is of addition of reaſons.

TO finde the aggregate of many reaſons giuen of one kinde, know that as (in this place) reaſons are nothing but the numbers that name of quantities giuen, what one quantitie is of an other: ſo the addition of the ſame reaſons, is to finde the number that nameth the reaſon betweene the quantities that are compounded of the quantities giuen, the one of the antecedents, the other of the conſequents, and ſo conſequently the reaſon demaunded, is the product of the numbers which name the reaſons giuen ſo taken and multiplied together, for that the numbers which name the reaſons are compared vnto 1, or 1 vnto the numbers which name the reaſons, &c,

Examples.

If $1\frac{1}{2}$ times, and $1\frac{1}{3}$ times be the reaſons giuen to be added together, their aggregate is $2\frac{1}{4}$ times, for $1\frac{1}{3}$ multiplied by $1\frac{1}{2}$ the product is $2\frac{1}{4}$. If $1\frac{1}{2}$ times $1\frac{1}{3}$ times and $1\frac{1}{2}$ times be the reaſons giuen to be added together, their aggregate is $3\frac{3}{8}$ times, for the ☌ of $1\frac{1}{2}$ is $3\frac{3}{8}$ &c: of all other.

If 2 times, $3\frac{3}{7}$ times and $3\frac{3}{7}$ times be the reaſons giuen to be added together, their aggregate is $24\frac{12}{17}$ times. If $\sqrt{3}$ 3 times & $\sqrt{3}$ 3 times be giuen to be added together, their aggregate is 3 times. If $\sqrt{3}$ 3 times $\sqrt{3}$ 3 times and $\sqrt{3}$ 3 times be giuen to be added together, their aggregate is $\sqrt{3}$ 27 times. If $\sqrt{3}$ 2 times and $\sqrt{\text{☌}}$ 3 times be giuen to be added together, their aggregate is $\sqrt{3}\sqrt{\text{☌}}$ 72 times &c: The like you muſt doe if the reaſons giuen to be added together, be in the leſſer inequalitie: as if ſub $1\frac{1}{2}$ times, ſub $1\frac{1}{3}$ times and ſub $2\frac{1}{4}$ times be giuen to be added togerher, their aggregate is ſub $6\frac{7}{8}$ times. If ſub $\sqrt{3}$ $1\frac{1}{2}$ times, ſub $\sqrt{\text{☌}}$ $1\frac{1}{3}$ times and ſub $2\frac{1}{4}$ times be giuen to be added together, their aggregate is ſub $\sqrt{3}\sqrt{\text{☌}}$ 77 $\frac{88\text{11}}{184\text{33}}$ times &c: but if ſome of the reaſons giuen to be added together, be in the reaſon of the greater inequalitie, and ſome in the reaſon of the leſſer inequalitie: then adde all the reaſons of the greater inequalitie giuen by themſelues, and all the reaſons of the leſſer inequalitie giuen by themſelues; then deuide the greater number

that

that nameth the reaſon of thoſe aggregates by the leſſer, and the quotient is the number that nameth the reaſon deſired, and taketh his name of greater or leſſer inequalitie, of the greater aggregate aforeſaid, that is of the deuident: but if the aforeſaid aggregates be equall, the reaſon demaunded is in equalitie. As if 1 $\frac{1}{2}$ times and ſub 1 $\frac{1}{2}$ times be giuen to be added together, their aggregate is in equalitie equall, or one time. If 1 $\frac{1}{3}$ times and ſub 2 $\frac{1}{3}$ times bee giuen to bee added together, their aggregate is ſub 1 $\frac{2}{7}$ times. If 3 times and ſub 2 times bee giuen to be added together, their aggregate is 1 $\frac{1}{2}$ times. If 1 $\frac{1}{4}$ times, 2 $\frac{2}{7}$ times and ſub 1 $\frac{1}{3}$ times be giuen to be added together, their aggregate is 2 $\frac{7}{16}$ times. If 1 $\frac{1}{5}$ times, 2 $\frac{1}{2}$ times, ſub 2 $\frac{1}{3}$ times and ſub 3 $\frac{3}{4}$ times be giuen to be added together, their aggregate is ſub 3 $\frac{169}{312}$ times &c.

The 25 Inſtruction is, of ſubſtractions of reaſons.

TWo reaſons of one kinde being giuen, to finde the reaſon remaining, when the one is ſubſtracted out of the other. Know that as addition of reaſons is effected by the compoſition of quantities, ſo ſubſtraction is made by reducing of quantities to the quantities whereof they are compounded. Therefore ſubſtraction of reaſons doth onely differ from addition of reaſons in this, that where you multiplied to make addition, there you muſt deuide to make ſubſtraction, and where you deuided to make addition, there you muſt multiply to make ſubſtraction, &c:

Examples.

☞ If 1 $\frac{1}{2}$ times be giuen to be abated out of 2 $\frac{1}{4}$ times, the remainer is 1 $\frac{1}{2}$ times, for 2 $\frac{1}{4}$ deuided by 1 $\frac{1}{2}$ the quotient is 1 $\frac{1}{2}$ &c: of all other. If 1 $\frac{1}{2}$ times be giuen to be abated out of 3 $\frac{1}{8}$ times, the remainer is 2 $\frac{1}{4}$ times. If $\sqrt{3}$ 3 times be giuen to be abated out of 3 times, the remainer is $\sqrt{3}$ 3 times. If $\sqrt{3}$ 2 times be giuen to be abated out of $\sqrt{c}$ 9 times, the remainer is $\sqrt{3}\,\sqrt{c}$ 10 $\frac{1}{2}$ times. If $\sqrt{3}$ 2 times be giuen to be abated out of $\sqrt{3}\,\sqrt{c}$ 72 times, the remainer is $\sqrt{c}$ 3 times &c. If ſub 1 $\frac{1}{2}$ times be

giuen to be abated out of ſub 2 ½ times, the remainer is ſub 1 ½ times &c: If 1 ½ times be giuen to be abated out of 1 ½ times, the remainer is equalitie, &c: If three times be giuen to be abated out of 2 times, the remainer is ſub 1 ½ times, where note, that the remainer is altered from the greater inequalitie giuen, to the leſſer inequalitie, by reaſon that the greater reaſon giuen, is giuen to be ſubſtracted out of the leſſer &c: if ſub √ʒ 3 times be giuen to be ſubſtracted out of ſub √ʒ 2 times, the remainer is √ʒ 1 ½ times &c: where note that the remainer is altered from the leſſer inequalitie giuen to the greater &c: if 1 ½ times be giuen to be abated out of ſub 1 ½ times, the remainer is ſub 2 ½ times. If ſub 2 times be giuen to be abated out of 1 ½ times, the remainer is 3 times &c.

The 26 Inſtruction is, of multiplications of reaſons.

TO multiply the reaſon giuen, by a number giuen, the number that nameth the reaſon of the product, is the product of the number that nameth the reaſon giuen ſo many times taken and multiplied as the number giuen is vnits: which in effect is the aggregate of the ſo many times taking of the reaſon giuen and added together. By which it is manifeſt, that if the reaſon giuen, be to be multiplied by a fraction giuen: multiply the reaſon giuen by the numerator of the fraction giuen, and then the ſuch roote of that number that nameth the reaſon of that product, taken as the product of a roote ſo many times taken and multiplied as the denominator of the fraction giuen doth containe vnits, is the number that nameth the reaſon required &c.

Examples.

If 1 ½ times be giuen to be multiplied by 3, the number that nameth the reaſon of the product, is the cℭ of the number that nameth the reaſon giuen, and is 3 ⅜ times. If 1 ½ times be giuen to be multiplied by 2, the number that nameth the reaſon of the product is the ʒ of the number that nameth the reaſon giuen, and is 2 ¼ times &c: euer the product of the number that nameth

nameth the reaſon giuen, ſo many times taken and multiplied as the number giuen doth containe vnits &c. If $3\frac{1}{3}$ times be giuen to be multiplied by $\frac{1}{3}$ the number that nameth the reaſon of the product is the ce roote of the number that nameth the reaſon giuen, and is $1\frac{1}{3}$ times. If $2\frac{1}{4}$ times be giuen to be multiplied by $\frac{1}{2}$, the number that nameth the reaſon of the product is the z roote of the number that nameth the reaſon giuen, and is $1\frac{1}{2}$ times. If $1\frac{1}{3}$ times be giuen to be multiplied by $\frac{1}{2}$, the number that nameth the reaſon of the product is the zz roote of the number that nameth the reaſon giuen, and is $\sqrt{zz}\ 1\frac{1}{3}$ times &c. If $3\frac{1}{8}$ times be giuen to be multiplied by $\frac{2}{3}$, the product is $2\frac{1}{4}$ times. If $1\frac{1}{8}$ times be giuen to be multiplied by $\frac{2}{3}$, the product is $\sqrt{ſ8}\ 3\frac{1}{8}$ times &c. If 2 times be giuen to be multiplied by $3\frac{1}{2}$, the product is 8 times and $\sqrt{z}\ 2$ times, whoſe aggregate is $\sqrt{z}\ 128$ times. If 3 times be giuen to be multiplied by $2\frac{1}{4}$, the product is 9 times & $\sqrt{zz}\ 27$ times, whoſe aggregate is $\sqrt{zz}\ 177147$ times &c. If $\sqrt{z}\ 3$ times be giuen to be multiplied by 3, the product is $\sqrt{z}\ 27$ times. If $\sqrt{ce}\ 2$ times be giuen to be multiplied by $\frac{1}{2}$, the product is $\sqrt{zce}\ 2$ times. If $\sqrt{ce}\ 3$ times be giuen to be multiplied by $\frac{2}{3}$, the product is $\sqrt{ce}\ 9$ times, &c.

The 27 Inſtruction is, of deuiſions of reaſons.

TO deuide the reaſon giuen by an other reaſon giuen of the ſame kinde: is to finde how many times the one is contained in the other. Therefore ſo many times as the number that nameth the reaſon of the deuider, muſt be taken and multiplied to produce the number that nameth the reaſon of the deuident, ſo many times the deuider is contained in the deuident. If the deuider be a greater reaſon then the deuident, then the quotient is a fraction, and the operation is the conuerſe of the aforeſaid. To deuide the reaſon giuen by a number giuen: is to finde a reaſon that is contained in the reaſon giuen, ſo many times as the number giuen doth containe vnits: therefore the ſuch roote of the number that nameth the reaſon giuen, taken as the product of a reaſon ſo many times taken & multiplied as the deuider doth containe vnits, is the nūber that nameth the reaſon required &c.

Examples.

If $2\frac{1}{4}$ times be giuen to be deuided by $1\frac{1}{2}$ times, the quotient is 2. For $1\frac{1}{2}$ times $1\frac{1}{2}$ is $2\frac{1}{4}$ &c, of all other. If $3\frac{3}{8}$ times be giuen to be deuided by $1\frac{1}{2}$ times, the quotient is 3. If 32 times be giuen to be deuided by 2 times, the quotient is 5 &c. If $1\frac{1}{2}$ times be giuen to be deuided by $2\frac{1}{4}$ times, the quotient is $\frac{1}{2}$. If $1\frac{1}{2}$ times be giuen to be deuided by $3\frac{3}{8}$ times, the quotient is $\frac{1}{3}$. If 2 times be giuen to be deuided by 32 times, the quotient is $\frac{1}{2}$ &c. If $2\frac{1}{4}$ times be giuen to be deuided by 2, the quotient is $1\frac{1}{2}$ times. If $3\frac{3}{8}$ times be giuen to be deuided by 3, the quotient is $1\frac{1}{2}$ times. If 32 times be giuen to be deuided by 5, the quotient is 2 times &c. If $3\frac{3}{8}$ times be giuen to be deuided by $\frac{3}{4}$, multiplie the reason giuen first by 4 the denominator of the fraction giuen, then deuide that product by 3 the numerator of the fraction giuen, and the quotient is $5\frac{1}{16}$ times. Or deuide the reason giuen, by the numerator of the fraction giuen, and multiply that quotient by the denominator of the fraction giuen, and then that product is also the quotient desired. If $2\frac{1}{4}$ times be giuen to be deuided by $\frac{3}{4}$, the quotient is $3\frac{3}{8}$ times &c. If $2\frac{1}{4}$ times be giuen to be deuided by $3\frac{3}{8}$ times. For that the lesser reason giuen is to be deuided by the greater, and that it is not a part thereof as is euident, therefore deuide the greater reason giuen by the lesser; then againe the deuider of that deuision by the remainer of the same, and againe the deuider of that deuision by the remainer of the same &c: vntill there be no remainer, then by that reason which is the last such deuider, deuide seuerally both the reasons giuen, the quotient of the deuident is numerator and the quotient of the deuider is denominator of the fraction required: as in this example, $3\frac{3}{8}$ times deuided by $2\frac{1}{4}$ times, the quotient is 1, the remainer $1\frac{1}{2}$ times, thē $2\frac{1}{4}$ times deuided by $1\frac{1}{2}$ times, the quotient is 2 and nothing remaining, therefore deuide $2\frac{1}{4}$ times, and $3\frac{3}{8}$ times, each by $1\frac{1}{2}$ times, their quotients are 2 and 3, therfore the quotient desired is $\frac{2}{3}$. If $3\frac{3}{8}$ times be giuen to be deuided by $2\frac{1}{4}$ times, the quotient is $1\frac{1}{2}$. If $3\frac{3}{8}$ times be giuen to be deuided by $5\frac{1}{16}$ times, the quotient is $\frac{3}{4}$. If $5\frac{1}{16}$ times be giuen to be deuided by $3\frac{3}{8}$ times, the quotient is $1\frac{1}{3}$. If $17\frac{11}{137}$ times be

giuen

giuen to be deuided by $3\frac{1}{8}$ times, the quotient is $2\frac{1}{3}$. If $17\frac{11}{125}$ times be giuen to be deuided by $2\frac{1}{7}$, reduce the deuider in a fraction thus $\frac{7}{3}$ &c: and the quotient is $3\frac{1}{8}$ times &c.

The 28 Inſtruction is to finde betweene two numbers giuen of one kinde, ſo many numbers middle proportionals as are required.

THe ſuch roote of the number that nameth the reaſon betweene two numbers giuen, taken as the product of ſo many multiplications as there are middle proportionall numbers required to be found betweene them, is the number that nameth the reaſon that is beeweene the numbers demaunded: therefore continue your progreſſion betweene the two numbers giuen in that reaſon ſo found, and you haue your deſire, &c.

Examples.

If 6 and 18 be giuen, to haue 5 numbers middell proportionals to be found betweene them: their reaſon is ſub 3 times, the ȝ cc roote whereof is ſub √ȝ cc 3 times, the reaſon betweene the numbers demaunded, according to which reaſon continue your progreſſion betweene the numbers giuen thus, 6. √ȝ cc 139968. √cc 648. √ȝ 108. √cc 1944. √ȝ cc 11337408. 18, and thoſe are the 5 numbers demaunded. If 6 and 12 be giuen to haue 2 numbers middell proportionals to be found betweene them: their reaſon is ſub 2 times, the cc root wherof is ſub √cc 2 times, the reaſon between the nũbers demanded, according to which reaſon continue your progreſſiõ betweene the nũbers giuen thus: 6. √cc 432. √cc 864. 12 & thoſe are the two numbers required. If 5 & 8 be giuẽ to haue one number middel proportionall to be found between them: multiplie the two numbers giuen together, their product is 40, therefore √ȝ 40 is the number deſired. The progreſſion is 5. √ȝ 40. 8 or working as is aforeſaid, their reaſon is ſub $1\frac{7}{25}$ times, the ȝ roote whereof is ſub √ȝ $1\frac{7}{25}$ times, then the progreſſion is the ſame, that is aforeſaid, and ſo in effect are both the operations, &c.

The 29 Inſtruction is, of extraction of rootes in ſurde numbers.

ANy ſurde number being giuen, to extract his ſuch roote as is required: is no more but to place before the ſurd number giuen, the caracter , which ſheweth what product of multiplications the ſurde number giuen is taken to be, with this caracter ✓ placed firſt before it: all which by the 4 inſtruction of this booke is manifeſted. But if the ſurde number giuen be the ſuch product produced of a ſurde number, ſo many times taken and multiplied &c: and that it is required to finde that ſuch roote &c: you muſt haue aſwell regarde to the 2 and 28 Inſtruction of this booke, as conſideration to the ſuch multiplications of the ſuch ſurde numbers, being placed in thoſe perticulers, which are agreeable to the intent of the aforeſaid inſtructions. And for your more perceiuerance, we haue placed heere the figures and caracters of the ſuch operations of the ȝ . ȼ . ȝȝ and ſȣ of 4 + ✓ȝ 2

The ȝ of 4 + ✓ȝ 2 placed in his parts.

16 . ✓ȝ 32 . 2
✓ȝ 32 .

The ȼ of 4 + ✓ȝ 2 placed in his parts.

64 . ✓ȝ 512 . 8 . ✓ȝ 8
✓ȝ 512 . 8
✓ȝ 512 . 8

The ȝȝ of 4 + ✓ȝ 2 placed in his parts.

256 . ✓ȝ 8192 . 32 . ✓ȝ 128 . 4
✓ȝ 8192 . 32 . ✓ȝ 128
✓ȝ 8192 . 32 . ✓ȝ 128
✓ȝ 8192 . 32 . ✓ȝ 128
32
32

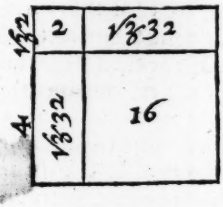
And the ſȣ of 4 + ✓ȝ 2 placed in his parts, is as hereafter you ſee on the next page following.

his

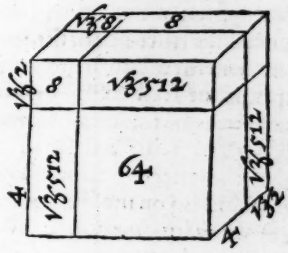
his ſ8

1024 . √ʒ 131072 . 128 . √ʒ 1248 . 16 . √ʒ 32
√ʒ 131072 . 128 . √ʒ 1248 . 16
√ʒ 131072 . 128 . √ʒ 1248 . 16
√ʒ 131072 . 128 . √ʒ 1248 . 16
√ʒ 131072 . 128 . √ʒ 1248 . 16
128 . √ʒ 1248
128 . √ʒ 1248
128 . √ʒ 1248
128 . √ʒ 1248
128 . √ʒ 1248

The ſquare.



The cube.



for by them it dooth appeare that of ʒ binomes produced of a binome roote: if the firſt particuler of the binome giuen be parted in two ſuch partes, as multiplied together do produce the ʒ of the $\frac{1}{3}$ of the laſt particuler of the binome giuen, then the ſeuerall √ʒ of each of thoſe ſuch partes added together is the ze demanded. Of ce binomes produced of a binome ze, if the firſt particuler of the binome giuen, be parted in two ſuch parts, that the $\frac{1}{3}$ of his ſecond parte, be the leaſt of two middell proportionalles, falling betweene his firſt parte, and the quotient of the $\frac{1}{3}$ of his ſecond part, deuided by the √ʒ of the quotient of his firſt part deuided by the $\frac{1}{3}$ of his ſecond part,

and that the aggregate of 3 times the greater middell proportionall added to the ſaid quotient ſo laſt found (betweene which and the firſt part of the firſt particular aforeſaid the two middell proportionals do fall) be equall to the ſecond particuler of the binome giuen, then the ſeuerall √c of each of the ſaid two numbers betweene which the two ſuch middell proportionals do fall, added together is the roote demaunded. And by more ſpeculatiue regarde had of c binomes, it is apparent, that the firſt and greater perticuler of the binome giuen, is the aggregat of the firſt or greater extreame of the progreſſion which we intend to finde, and three times the leaſt middell proportionall: and the other and leaſt perticuler of the binome giuen, is the aggregate of the other or leſſe extreame of the ſaid progreſſion and three times the greater middell proportionall: yet farther, that the ſquares of the particulers of the binome giuen, being abated the leſſer out of the greater, the c roote of the remainer, will ſhew how many times the c roote of the greater extreame the leaſt middell proportionall is leſſe then the greater extreame: and alſo how many times the c roote of the leſſer extreame, the greater middell proportionall is more then the leſſer extreame. The like inſight and ſpeculations in their kinds you muſt haue, to finde the rootes of ʒʒ . ſ8 or any other binome giuen. Whoſe opperations to the witty are apparent by the aforeſaid placing of the particulars of each multiplication, &c.

When the √c of the remainer is ſurde, then the binome giuen hath no binome for his roote

Examples.

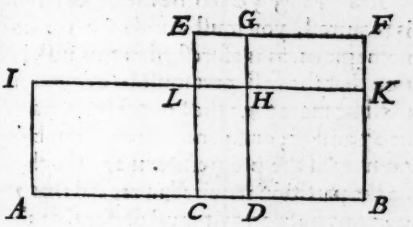
The ʒ roote of √ʒ 3, is √ʒʒ 3. The c roote of √ʒ 5, is √ʒc 5. The ʒ roote of 8 + √ʒ 2, is √ʒ . 8 + √ʒ 2. The c roote of √c 9 + 8, is √c . √c 9 + 8. The ʒʒ roote of √ʒ 17 + √ʒʒ 11, is √ʒʒ . √ʒ 17 + √ʒʒ 11 &c.

If 18 + √ʒ 128 be the ʒ of a binome, and his ʒ roote is required, the ½ of the laſt particuler is √ʒ 32, his ʒ is 2, and 18 cannot be parted or deuided into any two numbers which multiplied together produce 32, but onely in 16 and 2, therefore √ʒ 16 + √ʒ 2, that is 4 + √ʒ 2, is the roote demaunded.

manded. If $50+\sqrt{3}\,2501$ be the $\sqrt{3}$ of a binome, & his $\sqrt{3}$ root is required, the $\frac{1}{3}$ of the laſt particular is $\sqrt{3}\,5004$, his $\sqrt{3}$ is 504, now 50 deuided &c. is 36 and 14, therefore $6+\sqrt{3}\,14$, is the roote demanded.

If $\sqrt{3}\,32+\sqrt{3}\,24$ be the $\sqrt{3}$ of a binome, and his $\sqrt{3}$ roote is required, the $\frac{1}{3}$ of the laſt particular is $\sqrt{3}\,6$, his $\sqrt{3}$ is 6, now deuide $\sqrt{3}\,32$ in two parts &c. but becauſe $\sqrt{3}\,32$ is a ſurd number, reduce 6 alſo into $\sqrt{3}\,36$ like a ſurd number, & conſider which two ſurd numbers multiplied together wil produce $\sqrt{3}\,36$, and that being added together, are $\sqrt{3}\,32$. Firſt there are no two ſurd numbers which by their multiplication do produce $\sqrt{3}\,36$, but $\sqrt{3}\,18$ and $\sqrt{3}\,2$, or $\sqrt{3}\,12$ and $\sqrt{3}\,3$, and of theſe there is none whoſe aggregat is $\sqrt{3}\,32$ but $\sqrt{3}\,18$ and $\sqrt{3}\,2$, therefore $\sqrt{3}\sqrt{3}\,18+\sqrt{3}\sqrt{3}\,2$ is the root demanded.

If $\sqrt{3}\,18+4$ be the $\sqrt{3}$ of a binome, and his $\sqrt{3}$ root is required, the $\frac{1}{3}$ of the laſt particular is 2, his $\sqrt{3}$ is 4, reduced into the forme of a ſurd number is $\sqrt{3}\,16$, which cannot be produced but onely by theſe ſurd numbers $\sqrt{3}\,8$ and $\sqrt{3}\,2$, and their aggregat is alſo $\sqrt{3}\,18$, therefore $\sqrt{3}\sqrt{3}\,8+\sqrt{3}\sqrt{3}\,2$ is the roote demanded &c. But foraſmuch as the $\sqrt{3}$ roote of a binome may be required which hath no binome for his roote, or if it haue, yet there may be ſome difficultie to find the ſame by the reaſons aforeſayd. Therefore we will proceed to find thoſe ſuch roots or any other by the reaſon and helpe of the 5 propoſition of the 2 booke of Euclids Elements: where it is demonſtrated that if the line A B be deuided in two equall parts in the point C, and in 2 vnequall parts in the point D, that the parallelogram A H produced of A D in B D with the $\sqrt{3}$ L G (that is $\sqrt{3}$ of C D, which is the differēce betweene B C and B D) is as much, or equall to the $\sqrt{3}$ of B C (which is the $\frac{1}{3}$ of A B) Therefore if $4+\sqrt{3}\,60$ be giuen to haue his $\sqrt{3}$ root extracted, take the line



A B for the firſt particular, the line A D for his firſt part, the line B D for his other part, and the parallelogram A H for the ☐ of the ½ of the laſt particular giuen, for that it is produced of the lines A D in B D, that is of A D the firſt part of the firſt particular multiplied by B D the ſecond part of the firſt particular giuen. For the attaining to which ſayd parts, deuide the line A B that is 8 in two equall parts in the point C, then B C is 4, his ☐ C F, is 16, from which abate the parallelogram A H, the ☐ of the ½ of the laſt particular giuen, which is 15, remaineth L G 1 which is the ☐ of C D, whoſe ☐ root is 1 for C D now C D 1 added to A C 4 (the ½ of the firſt particular) is A D 5. Againe C D 1 abated out of B C 4 (the ½ of the firſt particular) remaineth B D 3. Therfore $\sqrt{☐}\,5+\sqrt{☐}\,3$ is the root demanded.

If $6+\sqrt{☐}\,60$ be giuen to haue his ☐ roote extracted, firſt alter the poſition of your binome giuen, and place the greater particular formoſt thus, $\sqrt{☐}\,60+6$, then the ½ of the firſt particular is $\sqrt{☐}\,15$, his ☐ is 15, the ½ of the laſt particular is 3, his ☐ is 9, take 9 out of 15, remaineth 6, his ☐ roote is $\sqrt{☐}\,6$, which $\sqrt{☐}\,6$ adde to $\sqrt{☐}\,15$, the aggregat is $\sqrt{☐}\,15+\sqrt{☐}\,6$, and the ſame $\sqrt{☐}\,6$ abate out of $\sqrt{☐}\,15$ the remainer is $\sqrt{☐}\,15-\sqrt{☐}\,6$, therefore $\sqrt{☐}\,.\,\sqrt{☐}\,15+\sqrt{☐}\,6 : +\sqrt{☐}\,.\,\sqrt{☐}\,15-\sqrt{☐}\,6$ is the roote demanded &c The operations to find the ☐ roote of the binome reſidues, is the ſame that the binomes haue: but that the ſeuerall ☐ rootes of the parts of the firſt particular muſt be ſubſtracted the one from the other, according as the reſidue giuen giueth cauſe to be done &c.

If $88+\sqrt{☐}\,5000$ be the ☌ of a binome, and his ☌ roote is required, you muſt deuide 88 the firſt particular of the binome giuen, as is aforeſayd in this inſtruction, with conſideration that the firſt part muſt be a ☌ number, and againe that it muſt be more then the ½ of the other part &c. ſo deuided it muſt be 27 and 51; or 64 and 24: now if it be 27 and 51, then the firſt number of the progreſſion is 27, the third 17, and that can not be the partition required, in regard that 17 doth not meaſure 27 without fraction as it muſt do, for that the binome giuē is without fractiō. If it be 64 and 24, then the firſt number of the progreſſion is 64, the third 8, the ſecond $\sqrt{☐}\,512$, and the fourth $\sqrt{☐}\,8$, whoſe ☐ is a cube number as it muſt be, beſide added to the

the product of 3 times the greater middle proportionall (that is of the second number) the aggregat is the last particular of the binome giuen: therfore √c64+√z'c8 that is 4+√z'2 is the roote demanded. But forasmuch as where the particulars of the binome giuen are great numbers &c. and which include many c numbers &c. there there may be some difficultie, to deuide the first particular giuen, as is aforesayd. Therfore by the farther speculation in this instruction mentioned, substract the squares of the particulars giuen, one frō other remaineth 2744, whose c roote is 14, to which adde such a number, as that the aggregat be a square number, and againe, that the z' of the surd particular giuen, being deuided by the number that is so to be added, the quotient may be a square number. That sayd number to be added in this place is 2, the aggregat 16. Therfore √z'16 +√z'2, that is 4+√z'2 is the roote desired.

If √z'50+7 be the c of a binome, and his c root is required, first alter the position of your binome giuē thus 7+√z'50, with this consideration, for that the least particular is formost, therfore the least number of your progression must be formost, by reason wherof the ⅓ of the second part of the particion of the first particular, must be the greatest middle proportionall, and greater then the first part of the particion of the first particular, &c. 7 so deuided it cannot be other then 1 & 6, the progression is 1 . √z'2 . 2 . √z'8 and √z'8 added to the product of 3 times √z'2 is the last particular &c. therefore √z'2+1 is the roote desired. Or substract 49 from 50, remaineth 1, whose c roote is 1, and by reason that the surde particular giuen is the greater, adde such a number to the sayd 1, that the z' of √z'50 being deuided by their aggregat, the quotient may be a square number. That their aggregat in this place is 2, therefore √z'2 +1 is the roote to be found.

If √z'243+√z'242 be a c binome, and his c roote is required, √z'243 cannot be deuided as aforesaid, but into √z'27 & √z'108, the progression is √z'27 . √z'18 . √z'12 . √z'8, and √z'8 added to the product of 3 times √z'18, is the last particular of the binome giuen. Therefore √z'3 +√z'2 is the roote demanded &c. Or substract 242 from 243, remaineth 1, whose c roote is 1, to which adde 2, the aggregat is 3. There-

fore $\sqrt{\mathcal{Z}}3+\sqrt{\mathcal{Z}}2$, is the roote desired.

If $10+\sqrt{\mathcal{Z}}50\frac{19}{27}$ be a ℭ binome, and his ℭ roote is required, in consideration of the fraction giuen, the first part of the particion of 10 must be a ℭ number, & the $\frac{1}{3}$ of the other part must fall in thirds, so that 10 cannot be deuided as aforesayd, but in 8 and 2, the progression is 8. $\sqrt{\mathcal{Z}}5\frac{1}{3}.\frac{2}{3}.\sqrt{\mathcal{Z}}\frac{1}{27}$ & the product of 3 times $\sqrt{\mathcal{Z}}5\frac{1}{3}$ added to $\sqrt{\mathcal{Z}}\frac{1}{27}$ is $\sqrt{\mathcal{Z}}50\frac{19}{27}$. Therfore $2+\sqrt{\mathcal{Z}}\frac{1}{3}$ is the roote desired. Or substract $50\frac{19}{27}$ from 100, remaineth $49\frac{8}{27}$ that is $\frac{1331}{27}$, whose ℭ roote is $\frac{11}{3}$, to which adde $\frac{1}{3}$, the aggregate is 4. Therefore $2+\sqrt{\mathcal{Z}}\frac{1}{3}$ is the roote desired.

If $8+\sqrt{\mathcal{Z}}66\frac{10}{27}$ be a cube binome, and his ℭ root is required, in consideration that 8 is the least particular &c. deuide it into 1 and 7, the progression is $1.\sqrt{\mathcal{Z}}2\frac{1}{3}.2\frac{1}{3}.\sqrt{\mathcal{Z}}12\frac{19}{27}$ and the product of 3 times $\sqrt{\mathcal{Z}}2\frac{1}{3}$ added to $\sqrt{\mathcal{Z}}12\frac{19}{27}$, is $\sqrt{\mathcal{Z}}66\frac{10}{27}$, therfore $\sqrt{\mathcal{Z}}2\frac{1}{3}+1$, is the roote demanded. Or substract 64 from $66\frac{10}{27}$, remaineth $2\frac{10}{27}$, that is $\frac{64}{27}$, whose ℭ roote is $\frac{4}{3}$, to which ad 1, the aggregat is $2\frac{1}{3}$. Therfore $\sqrt{\mathcal{Z}}2\frac{1}{3}+1$ is the root required.

If $12\frac{3}{8}+\sqrt{\mathcal{Z}}153\frac{1}{8}$ be a ℭ binome, and his ℭ roote is required: first examine which is the greater particular, and that is the first, which cannot be deuided &c. but into $3\frac{3}{8}$ and 9, in regard of the fraction giuen, the progression is $3\frac{3}{8}.\sqrt{\mathcal{Z}}10\frac{1}{8}.3.\sqrt{\mathcal{Z}}8$, & $\sqrt{\mathcal{Z}}8$ added to the product of 3 times $\sqrt{\mathcal{Z}}10\frac{1}{8}$, is $\sqrt{\mathcal{Z}}153\frac{1}{8}$, therefore $1\frac{1}{2}+\sqrt{\mathcal{Z}}2$ is the roote demanded &c. Or substract $153\frac{1}{8}$ from $153\frac{9}{64}$ remaineth $\frac{1}{64}$, whose ℭ roote is $\frac{1}{4}$, to which adde 2, the aggregat is $2\frac{1}{4}$. Therefore $1\frac{1}{2}+\sqrt{\mathcal{Z}}2$, is the roote desired, &c. The like considerations and speculations in their kinds you must haue to extract the $\mathcal{Z}\mathcal{Z}.\mathcal{Z}\mathcal{Z}$, $\mathcal{Z}\mathcal{Z}$, $\mathcal{Z}\mathcal{C}.\mathcal{Z}\mathcal{Z}$ or anie other roote of a binome giuen being produced of a binome root &c. and the same of binome residues in their kinds &c.

If $5+\sqrt{\mathcal{C}}324+\sqrt{\mathcal{C}}486$ be a ℭ trinome produced of a binome roote and his ℭ roote is required, then the $\frac{1}{3}$ of $\sqrt{\mathcal{C}}486$ is the greater middle proportionall, the $\frac{1}{3}$ of $\sqrt{\mathcal{C}}324$ the lesser middle proportionall, and 5 the aggregat of both the extremes added together, the progression is $3.\sqrt{\mathcal{C}}18.\sqrt{\mathcal{C}}12.2$. Therfore $\sqrt{\mathcal{C}}3+\sqrt{\mathcal{C}}2$ is the roote desired &c. The rest to auoid tediousnesse wee leaue to the discreete inuentions of the [illegible] ous &c.

The end of the third Booke.